APPENDICES

Appendix A

COMMON SYNCHRO PARAMETERS

Synchro Control Transmitters (CX)

TYPE DESIGNATION	FREQ Hz	PRIMARY (ROTOR) RATED VOLTS	NO LOAD INPUT volts	NO LOAD INPUT amps (Max)	NO LOAD INPUT watts (Max)	D.C. RESIST-ANCE (ohms)	SECONDARY (STATOR) NO LOAD OUTPUT volts	NO LOAD OUTPUT Phase lead (deg)	D.C. RESIST-ANCE (ohms)	NOMINAL IMPEDANCE Z_{ro} (ohms)	Z_{rs} (ohms)	Z_{so} (ohms)	Z_{ss} (ohms)	STATOR ERROR (Mins)	RESIDUAL Fund (mV)	RESIDUAL Total (mV)
26V 08CX4(B1)	400	26	26	0.111	0.95	60	11.8	13.0	19	77 + j270	137 + j39	17 + j49	—	10	20	40
26V 08CX4c	400	26	26	0.153	0.86	26	11.8	8.0	10	32 + j185	70 + j23	9 + j32	12.5 + j2.7	7	20	30
26V 11CX4c	400	26	26	0.130	0.56	21	11.8	4.5	20.6	34 + j265	51 + j21	7.7 + j45	8.7 + j3.2	7	12	19
11CX4c	400	115	115	0.031	0.61	343	90	4.5	300	-550 + j4070	725 + j307	330 + j2080	387 + j147	7	45	75
15CX4d	400	115	115	0.085	1.41	97	90	3.6	86	179 + j1400	217 + j125	100 + j775	112 + j63	6	32	60
15CX6b	60	115	115	0.056	2.4	550	90	15.0	470	628 + j2210	1170 + j299	367 + j1190	630 + j143	7	75	110
18CX4d	400	115	115	0.110	1.32	25	90	1.0	37	78 + j1210	78 + j81	52 + j598	40 + j39	6	40	60
18CX6c	60	115	115	0.040	1.11	559	90	10.0	666	605 + j3130	1380 + j451	510 + j1580	740 + j150	8	30	85
23CX4d	400	115	115	0.245	2.95	15.5	90	1.8	11.8	31 + j530	31 + j36	15 + j263	15.7 + j17.7	6	32	48
23CX6d	60	115	115	0.080	1.74	195	90	6.0	276	242 + j1650	462 + j150	211 + j954	319 + j62	8	30	60

Synchro Control Transformers (CT)

TYPE DESIGNATION	FREQ Hz	PRIMARY (STATOR) RATED VOLTS	NO LOAD INPUT volts	NO LOAD INPUT amps (Max)	NO LOAD INPUT watts (Max)	D.C. RESIST-ANCE (ohms)	SECONDARY (ROTOR) NO LOAD OUTPUT volts	NO LOAD OUTPUT Phase lead (deg)	VOLTAGE GRADIENT volts/deg	D.C. RESIST-ANCE (ohms)	NOMINAL IMPEDANCE Z_{ro} (ohms)	Z_{rs} (ohms)	Z_{so} (ohms)	Z_{ss} (ohms)	STATOR ERROR (Mins)	RESIDUAL Fund (mV)	RESIDUAL Total (mV)
26V 08CT4(B1)	400	11.8	10.2	0.137	0.47	28	22.5	13.5	0.39	145	173 + j564	253 + j104	25 + j93	—	10	30	60
26V 08CT4c	400	11.8	10.2	0.023	0.057	99	22.5	8.5	0.39	423	607 + j2900	800 + j300	100 + j506	140 + j53	7	25	30
26V 11CT4d	400	11.8	10.2	0.086	0.184	16.7	22.5	6.0	0.39	87	130 + j716	151 + j73.5	20 + j128	27 + j13.8	7	15	18
11CT4e	400	90	78	0.018	0.31	529	57.3	4.5	1.0	347	510 + j3020	535 + j302	700 + j4900	900 + j515	7	32	60
15CT4c	400	90	78	0.010	0.165	897	57.3	4.2	1.0	589	837 + j5170	943 + j589	1020 + j8330	1500 + j982	6	32	60
15CT6d	60	90	78	0.013	0.210	1300	57.3	9.5	1.0	900	970 + j3800	1430 + j409	1140 + j6240	2280 + j836	6	45	65
18CT4c	400	90	78	0.007	0.07	863	57.3	2.5	1.0	367	800 + j7770	745 + j782	1360 + j12600	1240 + j1250	6	20	30
18CT6d	60	90	78	0.017	0.45	2140	57.3	18.0	1.0	1030	1050 + j3280	1880 + j611	1690 + j4800	2830 + j848	6	25	45
23CT4c	400	90	78	0.0057	0.071	730	57.3	2.0	1.0	330	750 + j8570	660 + j812	1230 + j14300	1100 + j1360	6	20	45
23CT6d	60	90	78	0.0185	0.50	1830	57.3	14.0	1.0	800	883 + j3080	1500 + j512	1380 + j4790	2370 + j791	6	30	45

Synchro Control Differential Transmitters (CDX)

TYPE DESIGNATION	FREQ Hz	PRIMARY (STATOR) RATED VOLTS	NO LOAD INPUT volts	NO LOAD INPUT amps (Max)	NO LOAD INPUT watts (Max)	D.C. RESISTANCE (ohms)	SECONDARY (ROTOR) NO LOAD OUTPUT volts	NO LOAD OUTPUT Phase lead (deg)	D.C. RESISTANCE (ohms)	NOMINAL IMPEDANCE Z_{ro} (ohms)	Z_{rs} (ohms)	Z_{so} (ohms)	Z_{ss} (ohms)	ERROR STATOR (Mins)	ERROR ROTOR (Mins)	RESIDUAL Fund (mV)	RESIDUAL Total (mV)
26V 08CDX4(B1)	400	11.8	10.2	0.200	0.800	19	11.8	13.0	34	—	—	20 + j56	—	10	10	30	60
26V 08CDX4c	400	11.8	10.2	0.108	0.300	24	11.8	9.5	36	33 + j124	46 + j14	24 + j108	39 + j14	7	7	20	30
26V 11CDX4c	400	11.8	10.2	0.150	0.340	10.6	11.8	5.7	16.6	17.6 + j86	20.7 + j8.7	12.2 + j75	17.5 + j8.5	7	7	17	26
11CDX4b	400	90	78	0.049	0.730	191	90	4.7	446	450 + j1930	487 + j200	242 + j1690	421 + j211	7	7	60	90
15CDX4d	400	90	78	0.090	1.340	108	90	5.2	139	159 + j1060	190 + j125	129 + j917	164 + j111	6	6	32	60
15CDX6c	60	90	78	0.038	0.630	515	90	10.0	960	780 + j2625	1114 + j270	435 + j2270	930 + j880	7	7	60	100
18CDX4c	400	90	78	0.128	1.210	47	90	3.0	46	65 + j669	72 + j71	63 + j623	65 + j64	6	6	40	75
18CDX6d	60	90	78	0.052	1.450	599	90	17.0	897	717 + j1850	1130 + j315	465 + j1490	885 + j308	7	7	60	100
23CDX4c	400	90	78	0.285	2.900	18	90	3.0	19	26 + j310	27 + j30	24 + j280	24.7 + j27.2	7	7	30	60
23CDX6c	60	90	78	0.090	1.820	255	90	11.0	315	—	453 + j147	214 + j947	—	8	8	40	65

Synchro Torque Transmitters (TX)

TYPE DESIGNATION	FREQ Hz	PRIMARY (ROTOR) RATED VOLTS	NO LOAD INPUT volts	NO LOAD INPUT amps (Max)	NO LOAD INPUT watts (Max)	D.C. RESISTANCE (ohms)	SECONDARY (STATOR) NO LOAD OUTPUT volts	NO LOAD OUTPUT Phase lead (deg)	D.C. RESISTANCE (ohms)	NOMINAL IMPEDANCE Z_{ro} (ohms)	Z_{rs} (ohms)	Z_{so} (ohms)	Z_{ss} (ohms)	ERROR STATOR (Mins)	RESIDUAL Fund (mV)	RESIDUAL Total (mV)	MINIMUM TORQUE GRADIENT per degree per gm.cm.	MAXIMUM CONTINUOUS TORQUE (gm.cm)	MAXIMUM CONTINUOUS DISPLACEMENT (deg)	PULL OUT TORQUE (gm.cm)
26V 11TX4c	400	26	26	0.280	1.00	7.8	11.8	3.8	2.8	13.7 + j114	19.4 + j8.7	3.1 + j19.4	3.3 + j1.3	7	—	—	0.55	25	38	40
11TX4b	400	115	115	0.060	1.08	163	90	6.0	148	285 + j2140	370 + j159	175 + j1090	191 + j76	7	—	—	0.61	25	38	40
15TX4b	400	115	115	0.200	3.10	37.5	90	2.5	40	100 + j955	96 + j68	65 + j493	48 + j33	6	120	220	2.2	22	10	85
18TX6a	60	115	115	0.105	4.00	245	90	14.0	300	335 + j1270	686 + j210	256 + j916	379 + j81	6	—	—	3.6	134	37	172
23TX4b	400	115	115	0.719	6.5	2.3	90	1.0	2.6	15.5 + j192	10.8 + j10.6	7.5 + j98	5.1 + j5.0	6	—	—	18.0	290	16	1380
23TX6b	60	115	115	0.230	6.00	75	90	7.0	103	96 + j738	210 + j63	78 + j445	106 + j24	8	—	—	8.6	475	44	700

Synchro Torque Receivers (TR)

TYPE DESIGNATION	FREQ Hz	PRIMARY (ROTOR) RATED VOLTS	NO LOAD INPUT			D.C. RESIST-ANCE (ohms)	SECONDARY (STATOR) NO LOAD OUTPUT		D.C. RESIST-ANCE (ohms)	NOMINAL IMPEDANCE				ERROR		MINIMUM TORQUE GRADIENT	MAXIMUM CONTINUOUS		PULL OUT TORQUE (gm.cm)	SYNCHRO-NISING TIME	
			volts	amps (Max)	watts (Max)		volts	Phase lead (deg)		Z_{ro} (ohms)	Z_{rs} (ohms)	Z_{so} (ohms)	Z_{ss} (ohms)	STATOR (Mins)	RECEIVER (Mins)	per degree per gm.cm.	TORQUE (gm.cm)	DISPLACEMENT (deg)		30° (secs)	175° (secs)
26V 11TR4b	400	26	26	0.280	1.10	7.8	11.8	3.8	2.8	13.7 + j114	19.4 + j8.7	3.1 + j19.4	3.3 + j1.3	7	60	0.61	25	38	40	1.5	2.5
11TR4b	400	115	115	0.060	1.08	163	90	6.0	148	285 + j2140	370 + j159	175 + j1090	191 + j76	7	60	0.61	25	38	40	1.5	2.5
15TR4c	400	115	115	0.19	3.40	37.5	90	2.5	40	100 + j995	96 + j68	65 + j493	48 + j33	6	45	2.2	22	10	85	1	2
15TR6a	60	115	115	0.20	3.10	400	90	13.0	340	502 + j2240	885 + j194	301 + j1400	509 + j106	6	45	2.2	70	33	95	1	2
18TR4b	400	115	115	0.43	4.00	9.5	90	1.5	10.5	25 + j370	25 + j25	16 + j180	12 + j12	5	45	7.2	104	12	455	1	2
23TR4b	400	115	115	0.719	6.5	2.3	90	1.0	2.6	15.5 + j192	10.8 + j10.6	7.5 + j98	5.1 + j5.0	6	45	18.0	290	16	1380	1	2

Appendix B

SYNCHRO AND RESOLVER MANUFACTURERS

Listed below are some of the manufacturers of synchros and resolvers classified according to country.

France

Precilec
48 Rue D'Alesia
75014 Paris
Tel. 707-69-39

Thomson CSF
52 Rue Guynemer / BP 28
92132 Issy-Les-Moulineaux
Tel. 554-95-15

Sagem
6 Avenue D'Iena
75783 Paris
Cedex 16
Tel. 723-54-55
Telex. 611-890 F

Germany

Siemens AG
UBK SK RK
Hoffmanstrasse 51
D-8000 Munchen 70
Tel. (089) 722 26959
Telex. 5288225

Japan

Tamagawa Seiki Co. Ltd.
3-19-9 Shinkamata,
Ohta-Ku
Tokyo
Japan 144
Tel. 03-731-2131
Telex. 246 6166

Shinkoh Communication Industry Co. Ltd.
TOC Building
No. 7-22-17 Nishi Gotanda
Shinagawaku
Tokyo
Telex. 246 7435

Sweden

Jungner Marine AB
Hemvärnsgatan 13
Box 1102, S-171 22
SOLNA
Tel. 08-98 01 20
Telex. 17300

United Kingdom

Moore Reed and Company Ltd
Walworth Industrial Estate
Walworth
Andover
Hants SP10 5AB
Tel. (0264) 4155/4355
Telex. 47654

Muirhead Vactric Components Ltd
Garth Rd
Morden
Surrey SM4 4LL
Tel. (01) 337 6644
Telex. 27796

United States

American Electronics Inc
1600 E. Valencia Drive
Fullerton
CA 92631
Tel. (714) 871-3020
TWX. 910-592-1256

Bendix Corporation
Route 46
Teterboro
NJ 07608
Tel. (201) 288-2000
TWX. 201-288-4550

Clifton Precision
(Litton Systems Inc)
Marple at Broadway
Clifton Heights
PA 19018
Tel. (215) 622-1000
TWX 510-669-9782

The Singer Company
Kearfott Division
1150 McBride Avenue
Little falls
NJ 07424
Tel. (201) 256-4000
TWX 710-988-5700

Harowe Servo Controls Inc
Chester Pike West
Chester
PA 19380
Tel. (215) 692-2700

Tachtronics Instruments Inc
1500 North Front St
New Ulm:
Minnesota 56073
Tel. (507) 354-3105
TWX 910-565-2251

Transicoil Inc
Trooper Rd
Worcester
PA 19490
Tel. (215) 277-1300
TWX 510-660-0132

Appendix C

HARMONIC DISTORTION OF THE REFERENCE WAVEFORM

Common Signal Distortion

Tracking type Synchro to Digital converters are not sensitive to distortion of the reference waveform, very large distortions like 20% third harmonic will have negligible effect on the working of the converters. Distortion of the reference will alter the internal loop gain of the converter, but since the type 2 servo loop is employed in all the tracking converters very large loop changes can be tolerated without errors being caused.

The following analysis shows the way in which distortion of the reference waveform is of no practical consequence.

If there is distortion on the reference the resolver form signals can be represented by:

$$\text{Sin}\,\phi \sum_{n=1}^{n=N} B_n \,\text{Sin}\,(n\omega t + \alpha_n)$$

and

$$\text{Cos}\,\phi \sum_{n=1}^{n=N} B_n \,\text{Sin}\,(n\omega t + \alpha_n)$$

where ϕ is the resolver angle, $\omega = 2\pi f$, where f is the reference fundamental frequency with B_n and α_n as the harmonic amplitudes and phases.

The operation of the tracking control loop is to multiply the resolver form signals by Cos θ and Sin θ respectively, where θ is the RDC output angle. They are then subtracted and the result is applied to a phase sensitive detector to produce the control loop error signal. The error signal is reduced to zero by the action of the control loop. Carrying out these operations in steps we have:

$$\epsilon_1 = \sum_{n=1}^{n=N} B_n \,\text{Sin}\,(n\omega t + \alpha_n)\,(\text{Sin}\,\phi\,\text{Cos}\,\theta - \text{Cos}\,\phi\,\text{Sin}\,\theta)$$

$$\epsilon_1 = \sum_{n=1}^{n=N} B_n \,\text{Sin}\,(n\omega t + \alpha_n)\,\text{Sin}\,(\phi - \theta)$$

where ϵ_1 is the error signal before the phase sensitive detector.

Then:

$$\epsilon = \text{Sin}\,(\phi - \theta) \sum_{n=1}^{n=N} B_n \int_{\omega t=0}^{\omega t=\pi} \text{Sin}\,(n\omega t + \alpha_n)\,dt$$

where ϵ is the error after the phase sensitive detector and integrator.

The summation part of the above equation ie.

$$\sum_{n=1}^{n=N} B_n \int_{\omega=0}^{\omega=\pi} \text{Sin}\,(n\omega t + \alpha_n)\,dt$$

is just a constant, it will change in value according to the harmonic content but the important point is that this gives rise only to a change of loop gain and for the type 2 loop no errors will be caused by very large changes in this factor.

Differential Distortion

While distortion of the reference waveform is of little consequence, since it occurs on *both* the sine and cosine channels, distortion of one channel only has a very different effect. In practice there is no reason why the carrier on the sine channel should be distorted differently from that of the cosine channel. It could be that amplifiers giving distortion are being used in which case it is worth knowing the effect of the differential distortion. Differential distortion does produce errors.

The following simple analysis shows the effect of 1% third harmonic (in phase with the carrier at 0°) added to the sine input with the cosine input undistorted.

Let the input signal be:

$$\text{Sin } \omega t \text{ Sin } \phi + \text{K Sin } 3\omega t \text{ Sin } \phi \quad \text{(Sine input)}$$

$$\text{Sin } \omega t \text{ Cos } \phi \quad \text{(Cosine input)}$$

As before the operation of the converter is to multiply the sine input by Cos θ and to multiply the cosine input by Sin θ, to subtract them, pass them through a phase sensitive detector and integrate the output to produce the error signal. Fig. C-1 shows the system and the equations for the voltages at the different points.

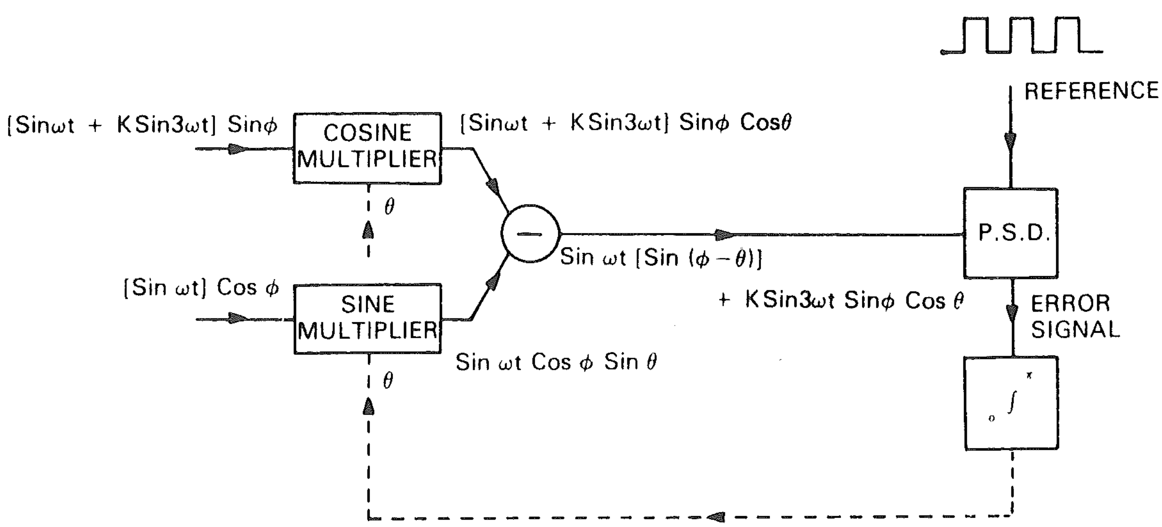

Fig. C-1 The effect of differential distortion on Sine Channel only.

(1) Assume $\theta \approx \phi$ due to the feedback.

(2) The error is caused by the second term of the output from the subtractor and for $\theta \approx \phi$, Sin ϕ Cos θ has a maximum of 0.5 for $\theta \approx \phi = 45°$.

(3) For $\theta \approx \phi = 45°$, the signal into the PSD is:

$$\text{Sin } \omega t \, [\text{Sin } (\phi - \theta)] + 0.5 \text{ K Sin } 3\omega t$$

(4) The output from the PSD is integrated and reduced to zero by the control loop.

(5) To simulate the effect of the PSD, integration is carried out only over one half period of the carrier. Due to the phase reversal of the PSD the other half period will be the same.

Writing $\phi - \theta = \epsilon$

$$\text{Sin } \epsilon \int_0^{\pi} \text{Sin } \omega t \, d\omega t + 0.5 \text{ K} \int_0^{\theta} \text{Sin } 3\omega t \, d\omega t = 0$$

or

$$\mathrm{Sin}\,\epsilon\,[\mathrm{Cos}\,\omega t]_{\omega t=0}^{\omega t=\pi} + 0.5K\,[\tfrac{1}{3}\,\mathrm{Cos}\,3\omega t]_{\omega t=0}^{\omega t=\pi} = 0$$

which gives:

$$2\,\mathrm{Sin}\,\epsilon + \frac{K}{3} = 0$$

and for ϵ in radians and ϵ small

$$\epsilon = -\frac{K}{6}$$

For example if $K = 0.01$ (1% third harmonic)

$$\epsilon^\circ = \frac{0.01}{6} \times 57 = 0.095^\circ$$

The conclusion here then is that differential harmonic distortion does have a considerable effect on the accuracy of the converter. Fortunately the areas where it is likely to occur are within the converters themselves, and care has been taken in the design to avoid errors due to this cause.

Appendix D

SPEED VOLTAGES IN RESOLVERS AND SYNCHROS

Introduction

In many application areas for synchros and synchro converters the synchros are required to generate signals from shafts rotating at high speeds. The requirement for synchros working at high speeds occurs particularly in geared systems, for example antenna systems where step up ratios of 36:1 are quite common. Generally the scanning rates are increasing, aerials rotating at 1 revolution per second are now being used instead of 0.25 revolutions per second; with gearing the synchro speed might be 36 revolutions per second or higher. In a completely different area of application, machine tool control, Inductosyns are often used to provide the position feedback. Such a system is electrically equivalent to a resolver where one pitch, for example 2mm, is equivalent to 360° rotation of the resolver. Resolver to digital converters are often used in conjunction with inductosyn systems. The speeds required may be 10 meters per minute or greater. 10 meters per minute is equivalent to 83 revolutions per second. These two examples are cases where relatively high tracking rates are required and in such cases the synchro to digital converters must be able to give the required accuracy in the presence of the so called "speed voltages" which will now be explained.

Speed voltages

The explanations which follow will be in terms of resolver format signals but are directly transferable to synchro form by using the Scott connected transformer operations. What we are assuming is that given two systems (1) A synchro transmitter and (2) A resolver feeding into a theoretically perfect Scott connected transformer, an observer with access to the input shafts and output wires would not find any difference between the two systems. If we analyse the resolver system the results will therefore also apply to the synchro system.

Figure D-1 shows the resolver being considered, the rotor is driven from a low impedance voltage source e and is rotating at a constant velocity of α radians per second. ϕ_1 and ϕ_2 represent the fluxes linking the windings and e_1 and e_2 are the instantaneous voltages across them. The stator windings are taken to be unloaded which will be the case when the outputs are fed into resolver to digital converters.

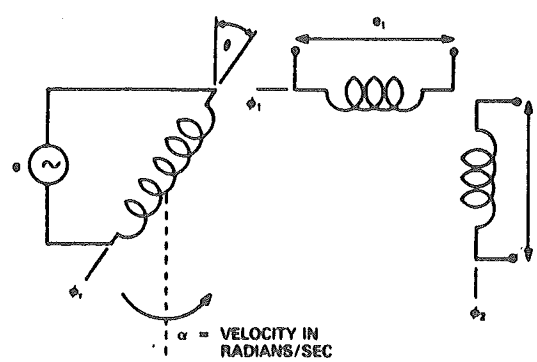

Fig. D-1 Resolver driven from a low impedance voltage source.

Taking $e = E \sin \omega t$ .... (1)

We shall first take the case of the impedance of the rotor as being purely reactive and equal to $X_R = \omega L$. The current which will flow in the rotor will be only due to e, the fact that it is rotating will not alter this current since there are no permanent magnets around and we have assumed that there are no currents in the stator coils. The current will lag on the voltage by 90° and is given by:

$$i = E/_{\omega L} \text{Cos}\, \omega t \quad \ldots (2)$$

This current will produce a flux in the rotor proportional to it and in phase with it, this flux is ϕ_r where:

$$\phi_r = KE/_{\omega L} \text{Cos}\, \omega t \quad \ldots (3)$$

The K in (3) will depend upon the iron, shape of the rotor, the number of turns etc. The geometry of the resolver is such that if the rotor has a magnetic flux ϕ_r and the resolver angle is set at θ one of the stator windings will have a flux ϕ_1 proportional to Sin θ and the other will have a flux ϕ_2 proportional to Cos θ, ie.

$$\begin{aligned} \phi_1 &= K_1 \phi_r \text{Sin}\, \theta \\ \phi_2 &= K_2 \phi_r \text{Cos}\, \theta \end{aligned} \quad \ldots (4)$$

and by design K_1 will be made equal to K_2.

In the case being considered θ is not a fixed angle, it is varying linearly with time, ie. $\theta = \alpha t + \theta_o$.

Substituting for ϕ_r using (3) and putting $\theta = \alpha t + \theta_o$ in the equations (4), also using $K_1 = K_2$ gives:

$$\begin{aligned} \phi_1 &= K_1 KE/_{\omega L} \text{Cos}\, \omega t\, \text{Sin}\, (\alpha t + \theta_o) \\ \phi_2 &= K_1 KE/_{\omega L} \text{Cos}\, \omega t\, \text{Cos}\, (\alpha t + \theta_o) \end{aligned} \quad \ldots (5)$$

The voltages induced in the two stator windings will be proportional to:

$d\phi_1/dt$ and $d\phi_2/dt$. Writing $K_1K/_L$ as A and differentiating with respect to t gives:

$$\begin{aligned} d\phi_1/dt &= AE\, \alpha/_\omega \text{Cos}\, \omega t\, \text{Cos}\, (\alpha t + \theta_o) - AE\, \text{Sin}\, \omega t\, \text{Sin}\, (\alpha t + \theta_o) \\ d\phi_2/dt &= -AE\, \alpha/_\omega \text{Cos}\, \omega t\, \text{Sin}\, (\alpha t + \theta_o) - AE\, \text{Sin}\, \omega t\, \text{Cos}\, (\alpha t + \theta_o) \end{aligned} \quad \ldots (6)$$

The equations (6) represent the voltages which will be produced in the two stator windings when the rotor is rotating at α radians per second.

If we put $\alpha = 0$ the voltages correspond with those of a stationary rotor at the angle θ_o, ie.

$$\begin{aligned} d\phi_1/dt &= -AE\, \text{Sin}\, \omega t\, \text{Sin}\, \theta_o \\ d\phi_2/dt &= -AE\, \text{Sin}\, \omega t\, \text{Cos}\, \theta_o \end{aligned} \quad \ldots (7)$$

The first terms in equation (6) are therefore extra voltages produced by the rotation — they are referred to as the "speed voltages". The important things to notice about them are that (1) The speed voltages are in quadrature with respect to the static voltages and (2) The maximum magnitude of the speed voltage is α/ω times the maximum static voltage.

Rejection of the speed voltages

Synchro and resolver to digital converters can use differing principles of operation, the so called "tracking" synchro to digital converter has many advantages over alternative methods and is almost universally used today. One of the advantages of the tracking type of converter is that the phase sensitive rectifier driven from the reference signal theoretically eliminates the effects of quadrature terms. In practice there are three requirements to be met for the elimination of the quadrature components. (1) The output of the phase sensitive detector must be integrated over an integral number of half periods of the carrier frequency. (2) The reference phase must be exactly correct, ie. in phase with the signal carrier. (3) The magnitude of the quadrature signals must not be such as to cause asymmetrical limiting in the phase sensitive detector or any amplifiers preceeding it.

The necessary integration or smoothing does not cause any problems, on the question of the reference phase being exactly correct there are problems which stem from the resolvers or synchros. The assumption made at the beginning of these notes was that the rotor of the resolver was purely reactive and that the stator coils were not loaded. The second requirement of effectively no load on the stator coils can be reasonably approached in practice but the rotor windings are not purely reactive, typical values will be K(1 + 5i) where K will vary according to the voltage. The effect of the resistive component is to shift the phase of the signal voltages relative to the reference by several degrees, in the case given by 11.2 degrees.

The effect of phase shift between signal and reference

The operation of the tracking RDC is to multiply the two voltages of equation (6) by Sin δ and Cos δ respectively and to subtract the results; then to carry out phase sensitive demodulation and integrate the result. A closed loop alters δ to reduce the integrated output of the phase sensitive detector to zero. If this operation is carried out on the static voltages given in equation (7), no error will result when the phase of the reference signal is shifted by up to 30 degrees or more. The reference phase shift will cause an effective change of loop gain but since the control loop is a type 2 system this does not give rise to errors. In the presence of the speed quadrature voltages this situation is no longer the case.

When the resolver is rotating at α radians per second and there is a phase shift of β between the signal and the reference the error signal due to the non velocity sensitive terms of (6) equal to AE Sin $(\alpha t + \theta_0 - \delta)$ will be modified by the effective gain of the phase sensitive detector to be:

$$-\text{Cos}\,\beta\ \text{AE Sin}\,(\alpha t + \theta_0 - \delta) \qquad (8)$$

but *in addition to* this the velocity sensitive terms of (6) will give rise to a voltage of:

$$\text{Sin}\,\beta.\text{AE}\alpha/_\omega\ \text{Cos}\,(\alpha t + \theta_0 - \delta) \qquad (9)$$

and the control loop will null the sum of the two terms, giving:

$$\text{Sin}\,\beta.\alpha/\omega\ \text{Cos}\,(\alpha t + \theta_0 - \delta) = \text{Cos}\,\beta\ \text{Sin}\,(\alpha t + \theta_0 - \delta) \qquad (10)$$

writing $\alpha t + \theta_0 - \delta$ as the error in angle ϵ and assuming that ϵ is small and β is small gives:

$$\beta.\alpha/_\omega = \epsilon \qquad (11)$$

The error due to "speed voltages" caused by a phase shift between the control signals and reference is given approximately by equation (11) where α and ω are the frequency of rotation and the carrier frequency in the same units and β is the reference to signal shift in the same units as ϵ, eg. if

$$\alpha = 30 \text{ radians/sec.}$$

$$\omega = 2500 \text{ radians/sec.}$$

and $\beta = 10$ degrees.

$$\frac{10 \times 30}{2500} = 0.12 \text{ degrees}$$

The voltages developed from a resolver rotating at α radians per second with a carrier frequency of ω radians per second are proportional to:

$$\alpha/\omega\ \text{Cos}\,\omega t.\ \text{Cos}\,(\alpha t + \theta_0) - \text{Sin}\,\omega t\ \text{Sin}\,(\alpha t + \theta_0),$$

and $-\alpha/\omega\ \text{Cos}\,\omega t.\ \text{Sin}\,(\alpha t + \theta_0) - \text{Sin}\,\omega t\ \text{Cos}\,(\alpha t + \theta_0)$

The first terms are the "speed" voltages where αt is the rotation speed in the same units as ω, ie. the carrier frequency. The angle of rotation is a linear function of time $\theta_0 + \alpha t$.

The mechanism of the SDC is to multiply the first voltage by Cos $\phi(t)$ and the second by Sin $\phi(t)$; to subtract the two terms and null the answer. $\phi(t)$ is the output digital angle as a function of time.

To simplify the mathematics we will ignore the transient situation and assume that $\phi(t) = \theta_0 + \alpha t + \epsilon$ where ϵ is a constant error, ie. we assume that the output is rotating at the same speed as the input where ϵ is the error to be determined. This assumption presupposes a long time constant smoothing between the phase sensitive detector and the output.

Multiplying the two terms by Cos $(\theta_0 + \alpha t + \epsilon)$ and Sin $(\theta_0 + \alpha t + \epsilon)$ respectively and subtracting to produce the error signal before the phase sensitive detector gives:

$$\alpha/\omega\ \text{Cos}\,\omega t.\ \text{Cos}\,(\alpha t + \theta_0)\ \text{Cos}\,(\theta_0 + \alpha t + \epsilon) + \alpha/\omega\ \text{Cos}\,\omega t\ \text{Sin}\,(\alpha t + \theta_0)\ \text{Sin}\,(\theta_0 + \alpha t + \epsilon),$$

$$-\text{Sin}\,\omega t\ \text{Sin}\,(\alpha t + \theta_0)\ \text{Cos}\,(\theta_0 + \alpha t + \epsilon) + \text{Sin}\,\omega t\ \text{Cos}\,(\alpha t + \theta_0)\ \text{Sin}\,(\theta_0 + \alpha t + \epsilon),$$

or $\alpha/\omega.$ Cos ωt Cos ϵ − Sin ωt Sin ϵ.

This error signal is integrated over half a period from $\omega t = \beta + \pi$ where β is the reference to signal phase shift to give

$$\alpha/\omega \text{ Cos } \epsilon \text{ Sin } \beta - \text{Sin } \epsilon \text{ Cos } \beta$$

equating this to zero and assuming ϵ and β small gives $\beta\alpha/\omega = \epsilon$.

Reduction of Signal to Reference phase angle

Most of the phase shift in a synchro or resolver which is not loaded on its stator outputs is due to the resistance of the rotor winding, to the extent that this is so, the phase shift can be improved by obtaining the reference voltage across a high Q inductance in series with the rotor. In the case of the reference transformer feeding into a high impedance amplifier the primary of the transformer can be put in series with the rotor energisation. The phase error between the signals and reference can be reduced to about 1 degree by this method as compared with 8 to 11 degrees being typical unloaded phase shifts. The need for reduction of the phase shift is to avoid speed errors, static errors are not caused by the phase shift.

In large installations where a standard reference is piped around the series technique may not be feasible, in this case individual phase correction will have to be used for example by putting a resistance in series with the reference transformer primary winding.

Synthetic reference

To reduce the effects of this phase shift an alternative reference which is derived from the signals plus the phase shifted reference can be used.

The word *improved* reference is used deliberately because the synthetic reference does not produce the ideal reference but it will generally be substantially better than say a reference shifted by 8 degrees. This point will be clearer when the method of deriving the Synthetic reference is explained.

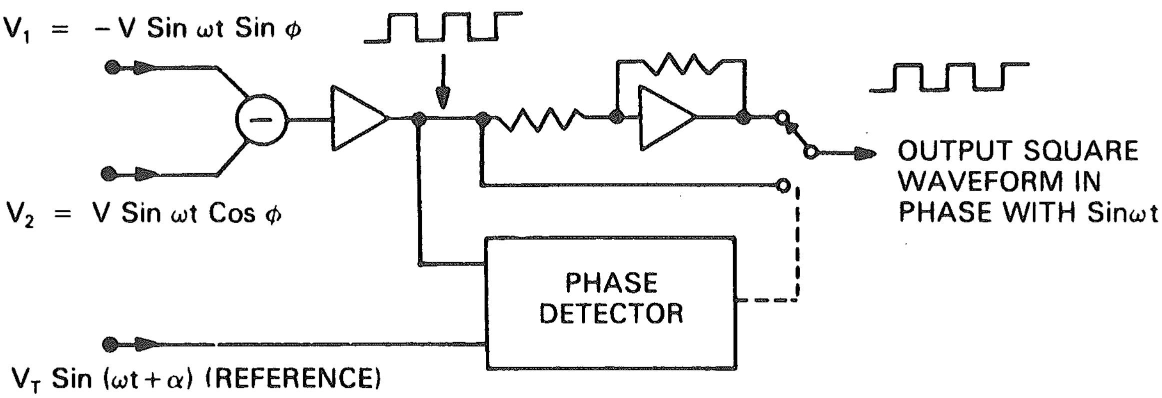

Fig. D-2 Principle of a synthetic reference.

Figure D-2 shows the method used. Two signal voltages V_1 and V_2 derived from the output of the quadrant select circuits of the SDC are used as inputs. These voltages are subtracted and limited to produce the output square waveform which is either taken directly or inverted according to the output from the phase comparator which uses the phase shifted reference and the limited signal as its inputs. The reason for saying that the output is an improved reference will be clear if we consider V_1 and V_2 to be other than the voltages shown.

One of the main applications for the use of a synthetic reference is when speed voltages are producing errors in the presence of a phase shifted reference. In these circumstances V_1 and V_2 will not be the voltages shown in figure D-2 but will be those given in equations (6), under the heading Speed Voltages, ie.:

$$-V_1 = \alpha/\omega \text{ Cos } \omega t \text{ Cos } (\alpha t + \theta) - \text{Sin } \omega t \text{ Sin } (\alpha t + \theta)$$

$$V_2 = \alpha/\omega \text{ Cos } \omega t \text{ Sin } (\alpha t + \theta) - \text{Sin } \omega t \text{ Cos } (\alpha t + \theta)$$

The reference required to completely eliminate speed errors is V sin ωt.

The synthetic reference produces a reference by subtracting V_1 and V_2 and the resultant does not have the phase of Sin ωt. The zero crossings of $V_1 - V_2$ which determine the square waveform output transitions do not coincide with the zero crossings of Sin ωt and therefore the synthetic reference obtained in this way is not equivalent to the ideal required. For reasonable values of α/ω the equivalent phase shift is considerably less than 8 degrees which may be on the actual reference and because of this, improvement in velocity errors will be obtained. The same type of argument follows when the Synthetic reference is used to reduce the effects of spurious quadrature residual signals, but in this case the improvement is usually very considerable. A detailed discussion would require knowledge of how the quadrature signal is caused and will therefore not be given here.

Appendix E

VECTOR ROTATION ALGORITHMS

In the synchro to digital and digital to synchro converters, the sine and cosine multiplication can be considered as an angular shifting operation, i.e. given an input vector represented by the components say x_1, y_1 the output from the sine and cosine multiplier system produces a new vector x_2 y_2 which is rotated in angle relative to the first by the angle θ which is applied to the multiplier. These operations are very similar to the CORDIC algorithm used in digital pocket calculators. Techniques very similar to the CORDIC rotation are used in some of the converters. In the following, the z transform is used to solve the generalised CORDIC equations given by J.S. Walther*. In the paper by J.S. Walther the solution to the equations is given without proof. The usefulness in the z transform method given below is that it leads us to an alternative set of equations which have advantages when applied to the synchro conversion problem.

In the paper by J.S. Walther, vectors $P_n = (x_n, y_n)$ and $P_{n+1} = (x_{n+1}, y_{n+1})$, which have a geometrical interpretation, are formed according to the itterative relations:

$$x_{n+1} = x_n + my_n\delta_n \qquad \ldots\ldots(1)$$

$$y_{n+1} = y_n - x_n\delta_n \qquad \ldots\ldots(2)$$

An additional equation

$$z_{n+1} = z_n + \alpha_n \qquad \ldots\ldots(3)$$

where

$$\alpha_n = m^{½} \tan^{-1}(m^{½}\delta_n) \qquad \ldots\ldots(4)$$

is used for accumulating the angles α_n.

Equations (1), (2), (3) and (4) form the basic Walther equations which reduce to the CORDIC equations for $m = 1$.

The equations have the solution

$$x_n = K\,[\,x_0 \operatorname{Cos}(\alpha m^{½}) + y_0 m^{½} \sin(\alpha m^{½})\,] \qquad \ldots\ldots(5)$$

$$y_n = K\,[\,y_0 \operatorname{Cos}(\alpha m^{½}) - x m^{½} \sin(\alpha m^{½})\,] \qquad \ldots\ldots(6)$$

where $K = \pi K_n$, $\qquad K_n = \surd(1 + m\delta_n^2)$

and $\alpha = \sum_{r=0}^{r=n-1} \alpha_n \qquad \alpha_n = \operatorname{Tan}^{-1}\delta_n \quad (m = 1)$

$$\alpha_n = \operatorname{Tan} n^{-1}\delta_n \quad (m = -1)$$

The values of $m = +1$, $m = 0$ and $m = -1$ give the trigonometric, linear and hyperbolic functions respectively.

*A Unified Algorithm for Elementary Functions, Spring Joint Computer Conference, 1971, J.S. Walther.

By trial equations (5) and (6) are found to satisfy equations (1) and (2). In what follows equations (5) and (6) will be obtained by a forward going process from equations (1) and (2), i.e. equations (1) and (2) will be solved by the use of the z transform. There are four things to be gained by this:

(1) The solution is obtained by forward going logical steps given (1) and (2), and the method can be used for other such equations.

(2) The method brings out the analogy between the difference equations and the second order differential equation which gives simple harmonic motion.

(3) It becomes obvious why $m = -1$ leads to the hyperbolic functions.

(4) The method leads us to alternative difference equations which are more useful for angular rotation of resolver form signals.

In the first algorithm δ_n is varied through a series of values to give rapid convergence, here we will first assume δ_n to be a constant and solve the difference equations for this condition. We will then show that δ can be changed in magnitude after any number of steps and the process continued.

Writing equations (1) and (2) as

$$x_{n+1} - x_n = m\, y_n\, \delta$$

$$y_{n+1} - y_n = -x_n\, \delta$$

or

$$\Delta x_n = m\, y_n\, \delta \qquad \ldots..(7)$$

$$\Delta y_n = -x_n\, \delta \qquad \ldots..(8)$$

Taking differences of equation (7) gives:

$$\Delta^2 x_n = m \Delta y_n \qquad \ldots..(9)$$

and substitution of y_n from (8) into (9) gives:

$$\Delta^2 x_n = -m\delta^2 x_n \qquad \ldots..(10)$$

By comparing equation (10) with the equivalent differential equation it is now obvious that the solution is likely to contain trigonometric or hyperbolic functions according to the sign of m.

We now proceed to solve equation (10) by using the z transform. Taking z transforms of both sides of equation (10) leads to:

$$(z-1)^2 X(z) - z\,[x_1 - x_0 + (z-1)x_0\,] = -m\delta^2 X(z) \qquad \ldots..(11)$$

This equation has been obtained by using line 4 and line 1 of the table of transforms in Fig. E-1. X (z) is the z transform of x_n.

Solving for X (z) gives:

$$X(z) = \frac{z\,[\,x_1 - x_0 + (z-1)\,x_0\,]}{(z-1)^2 + m\delta^2}$$

or

$$X(z) = \frac{z\,[\,x_1 - x_0 + (z-1)\,x_0\,]}{(z-1-i\delta\sqrt{m})(z-1+i\delta\sqrt{m})}$$

or using partial fractions

$$X(z) = \frac{z\,[x_1 - x_0 + (z-1)x_0\,]}{i\,2.\sqrt{m}\delta}\left(\frac{1}{[(z-1)-i\delta\sqrt{m}]} - \frac{1}{[(z-1)+i\delta\sqrt{m}]}\right) \qquad \ldots..(12)$$

Line	Function of n	z transform of the function of n
(1)	f (n)	$\sum_{n=0}^{n=\infty} f(n)z^{-n} = F(z)$
(2)	f(n − m)	$z^{-m} F(z) + z^{-m} \sum_{n=-1}^{n=-m} z^{-n} f(n)$
(3)	$\sum_{r=0}^{r=n} f(n-r)g(r)$	F(z)G(z)
(4)	$\Delta^s f(n)$ (sth difference of f(n))	$(z-1)^s F(z) - z[\Delta^{s-1} f(o) + (z-1)\Delta^{s-2} f(o) + (z-1)^2 \Delta^{s-3} f(o)t(o) \ldots (z-1)^{s-1} f(o)]$
(5)	$\left[\sum_{r=0}^{r=n}\right]^s f(n)$ *	$\frac{z^8}{(z-1)^3} F(z) + \left[\frac{z}{(z-1)} \left[\sum_0\right]^s f(n) + \frac{z^2}{(z-1)^2} \times \left[\sum_0\right]^{s-1} f(n) \ldots + \frac{'z^8}{(z-1)^3} \sum_0 (fn)\right]$
(6)	K − constant	$\frac{Kz}{(z-1)}$
(7)	$n^{(m)} = n(n-1)\ldots(n-m+1)$	$\frac{m!z}{(z-1)^{m+1}}$
(8)	a^n	$\frac{z}{z-a}$
(9)	$\sin \theta n$	$\frac{z \sin \theta}{z^2 - 2z \cos\theta + 1}$
(10)	$\cos \theta n$	$\frac{z(z-\cos\theta)}{z^2 - 2z \cos\theta + 1}$
(11)	Pulse of amplitude A at time m	$\frac{A}{z^m}$
(12)	* The symbol $\left[\sum_{r=0}^{r=n}\right]^s f(n)$ is equal to $\sum_{r_s=0}^{r_s=n} \sum_{r_{s-1}=0}^{r_{s-1}=r_s} \ldots \sum_{r=0}^{r=r_s} f(r)$ ie it is the sth summation of f(r). The symbol $\left[\sum_0\right]^s$ is the same as the above except the lower limit only is taken in the last summation.	

Fig. E-1 Table of z Transforms.

Taking inverse z transforms of equation (12) by using lines 8 and 2 in Fig. F-1 gives x_n as below:

$$x_n = \frac{x_1 - 2x_0}{i2\delta\sqrt{m}} [(1 + i\delta\sqrt{m})^n - (1 - i\delta\sqrt{m})^n]$$

$$+ \frac{x_0}{i2\delta\sqrt{m}} [(1 + i\delta\sqrt{m})^{n+1} - (1 - i\delta\sqrt{m})^{n+1}] \qquad \ldots\ldots(13)$$

The values of m of interest are m = + 1 and m = − 1. We shall proceed from here restricting m to be + 1 but the same method may be used for m = − 1 (the case of m = 0 has to be dealt with differently from before equation (12) because the roots are then equal).

Putting m = 1 in equation (13) gives

$$x_n = \frac{x_1 - 2x_0}{i2\delta} [(1+i\delta)^n - (1-i\delta)^n]$$

$$+ \frac{x_0}{i2\delta} [(1+i\delta)^{n+1} - (1-i\delta)^{n+1}] \quad .....(14)$$

Writing $1 + i\delta = re^{i\theta}$

and $1 - i\delta = re^{-i\theta}$

gives $r = \surd(1+\delta^2) \quad .....(15)$

$$\theta = \text{Tan}^{-1}\delta \quad .....(16)$$

and substitution of the equations (15) and (16) into equation (14) gives

$$x_n = \frac{(x_1 - 2x_0)}{\delta} r^n \text{Sin } n\theta + \frac{x_0}{\delta} r^{n+1} \text{Sin }(n+1)\theta \quad .....(17)$$

From equation (7), i.e.

$$x_{n+1} - x_n = m\, y_n$$

putting m = 1 and n = 0 gives

$$x_1 = x_0 + y_1$$

and substitution of this value for x_1 into equation (17) to obtain an equation for x_n in terms of x_0 and y_0 gives

$$x_n = \frac{(y_0\delta - x_0)}{\delta} r^n \text{Sin } n\theta + \frac{x_0}{\delta} r^{n+1} \text{Sin }(n+1)\theta$$

$$x_n = r^n [\, y_0 \text{Sin } n\theta - x_0 \frac{1}{\delta} (\text{Sin } n\theta - \text{Sin }(n+1)\theta)\,] \quad .....(18)$$

If $\delta = \tan\theta$ and $r = \surd(1+\delta^2)$ from (15) and (16) are substituted in the second term of (18) and use is made of:

$$\text{Sin }(n\theta + \theta) = \text{Sin }\theta \text{ Cos } n\theta + \text{Cos }\theta \text{ Sin } n\theta$$

we obtain

$$x_n = r^n [\, y_0 \text{Sin } n\theta + X_0 \text{Cos } n\theta \,] \quad .....(19)$$

Equation (19) corresponds to equation (5) for m = 1 and $\alpha = n\theta$, $K = r^n$.

Equation (19) is the solution to the difference equations (7) and (8) for x_n for m = 1 and for a fixed value of δ. Either by substitution for x_n in (19) or by the elimination of x_n and x_{n+1} from (7) and (8) to give an equation in y_{n+1} and y_n and solving this equation, y_n can be obtained in terms of x_0, y_0 in exactly the same way as x_n was obtained giving:

$$y_n = r^n [\, y_0 \text{Cos } n\theta - x_0 \text{Sin } n\theta \,] \quad .....(20)$$

Variable size angular steps

Having obtained a solution for fixed angular steps it is now necessary to find what happens if having proceeded through n iterations with a constant δ, its magnitude or its sign is changed and further steps are taken. Let the new step be $k\theta$ in size. After n steps of θ and 1 steps of $k\theta$ the solution will be

$$x_{n+1} = r'^{l} [y_0' \mathrm{Sin}\, lk\theta + x_0' \mathrm{Cos}\, lk\theta] \quad .....(21)$$

$$y_{n+1} = r'^{l} [y_0' \mathrm{Cos}\, lk\theta - x_0' \mathrm{Sin}\, lk\theta] \quad .....(22)$$

where the y_0', x_0' are the new initial conditions which equal y_n and x_n respectively and:

$$r' = \sqrt{(1+\delta')} \qquad \text{where } \delta' = \tan k\theta$$

Since $x_n = x_0'$ and $y_n = y_0'$, equations (19) and (20) can be substituted for x_0' and y_0' in equations (21) and (22). Carrying out the substitution and making use of the Sin $(A \pm B)$ formulae gives:

$$x_{n+l} = r^n r'^{l} [x_0 \mathrm{Cos}\,(n\theta + lk\theta) + y_0 \mathrm{Sin}\,(n\theta + lk\theta)] \quad .....(23)$$

$$y_{n+l} = r^n r'^{l} [y_0 \mathrm{Cos}\,(n\theta + lk\theta) - x_0 \mathrm{Sin}\,(n\theta + lk\theta)] \quad .....(24)$$

and this procedure can obviously be repeated for any number of steps any size and either sign.

An alternative algorithm

One of the problems in applying this algorithm in digital to resolver converters is caused by the fact that together with the angular shift there is a change in the vector length due to the fact that $r \neq 1$. r determines the amount by which the vector length increases at each step.

We now come to the justification for the lengthy method of solution of the equations. The CORDIC algorithm is equivalent to the solution of the difference equation (10) i.e.

$$\Delta^2 x_n = -\delta^2 x_n \quad .....(10)$$

and it has a solution which is divergent. This is in contrast with the differential equations

$$\frac{d^2y}{dt^2} = -cy \quad .....(25)$$

which has a solution which neither diverges or converges (no clamping). Equation (25) can be made to converge or diverge by the addition of a term proportional to $\frac{dy}{dt}$ and its magnitude and sign will determine the rate of divergence or convergence of the solution.

It seems reasonable therefore that the addition of a first difference term in equation (10) of a suitable magnitude and sign, will give an equation which has a solution with $r = 1$.

What we are seeking therefore is suitable values for A and B in equation (26) to give a sequence of values for x_n which represent the trigonometric functions with constant coefficients i.e. the radius should remain constant.

$$\Delta^2 x_n + A\Delta x_n + Bx_n = 0 \quad .....(26)$$

The z transform can again be used to solve equation (26). Taking z transforms using 4th line in Fig. E-1 gives:

$$(z-1)^2 X(z) - z[x_1 - x_0 + (z-1)x_0] + A[z-1)X(z) - zx_0] + BX(z) = 0$$

or

$$X(z) = \frac{z[x_1 - x_0 + (z-1)x_0] + Azx_0}{(z-1)^2 + A(z-1) + B}$$

$$X(z) = \frac{\text{Numerator}}{z^2 + (A-2)z + (B-A+1)} \quad .....(27)$$

The numerator of equation (27) is a polynomial in z and since powers of z in the numerator simply give rise to shifts in the ordinates the numerator need not be considered concerning questions of convergence or divergence of the solution. Whether the solutions of equation (27) diverge or converge depend on the position of the roots of the denominator. We can take a short cut however by referring to the table of z transforms; the 9th and 10th lines of Fig. E-1 show that the solution with a constant radius vector requires that:

$$A-2 = 2 \cos\theta$$

$$B-A+1 = 1$$

or

$$A = B = 2+2\cos\theta$$

and substituting these values in equation (26) gives

$$\Delta^2 x_n + [2+2\cos\theta]\,\Delta x_n + [2+2\cos\theta]\,x_n = 0 \qquad \ldots\ldots(28)$$

Equation (28) will generate trigonometric values x_n which have a constant radius. The equation will be more recognisable if it is put in terms of x_{n+2}, x_{n+1} and x_n.

$$\Delta^2 x_n = x_{n+2} - 2x_{n+1} + x_n$$

$$\Delta x_n = x_{n+1} - x_n$$

Substitution of these in (28) gives

$$x_{n+2} - 2x_{n+1} + x_n + (2+2\cos\theta)[x_{n+1} - x_n] + (2+2\cos\theta)x_n = 0$$

or

$$x_{n+2} - 2\cos\theta . x_{n+1} + x_n = 0 \qquad \ldots\ldots(29)$$

Equation (29) is the Chebyshev difference equation. Circuits based on this equation are used to rotate the resolver form voltages in some of the synchro converter products, that is the reason for including its derivation in these notes. Whilst the method of derivation of equation (29) implies that the x_n will generate the required angular shifts we have not so far solved the difference equation. Before doing so the more usual symbols for the Chebyshev equation will be used.

The Chebyshev difference equation is usually written

$$T_n(x) - 2xT_{n-1}(x) + T_{n-2}(x) = 0 \qquad \ldots\ldots(30)$$

The T in this equation is equivalent to the x in equation (29). The x in (30) is equivalent to Cos θ in (29). The n in (30) is equivalent to n + 2 in (29). To avoid confusion the variable x in (30) will be replaced by u giving the equation (31) which will be solved by the use of the z transform.

$$T_n(u) - 2uT_{n-1}(x) + T_{n-2}(x) = 0 \qquad \ldots\ldots(31)$$

The fact that T_n is written as a function of u is equivalent to saying that the x_n in (29) are functions of Cos θ, which they clearly are.

The solution of the Chebyshev difference equation

Since the difference equation holds for $n \geq 2$ the weighted summation from $n=2$ to $n=\infty$ will also be true. Therefore taking z transforms:

$$\sum_{n=2}^{n=\infty} z^{-n}[T_n(u) - 2uT_{n-1}(u) + T_{n-2}(u)] = 0$$

or
$$\sum_{n=0}^{n=\infty} z^{-n} T_n(u) - z^{-1} T_1(u) - T_0(u)$$
$$-2u \sum_{n=0}^{n=\infty} z^{-n} T_{n-1}(u) + 2uT_0(u)z^{-1} + 2uT_{-1}(u)$$
$$+ \sum_{n=0}^{n=\infty} z^{-n}T_{n-2}(u) - z^{-1}T_{-1}(u) - T_{-2}(u) = 0$$

putting $n-1=m$, $n-2=s$ gives:

$$\sum_{n=0}^{n=\infty} z^{-n} T_n(u) - z^{-1} T_1(u) - T_0(u)$$
$$= 2uz^{-1} \sum_{m=-1}^{m=\infty} z^{-m} T_m(u) + 2uT_0(u)z^{-1} + 2uT_{-1}(u)$$
$$+z^{-2} \sum_{s=-2}^{s=\infty} z^{-s} T_s(u) - z^{-1} T_{-1}(u) - T_{-2}(u) = 0$$

or
$$\sum_{n=0}^{n=\infty} z^{-n} T_n(u) - z^{-1} T_1(u) - T_0(u)$$
$$=2uz^{-1} \sum_{m=-1}^{m=\infty} z^{-m} T_m(u) - 2uz^{-2} T_{-1}(u) + 2uT_0(u)z^{-1} + 2uT_{-1}(u)$$
$$+z^{-2} \sum_{s=0}^{s=\infty} z^{-s} T_s(u) - T_{-2}(u) = z^{-1} T_{-1}(u) - z^{-1} T_{-1}(u) - T_{-2}(u) = 0$$

or on substituting $F(z,u) = \sum_{n=0}^{n=\infty} z^{-n} T_n(u)$

$$0 = F(z,u)[1-2uz^{-1}+z^{-2}] + [2uz^{-1}-1]T_0(u) - z^{-1}T_1(u)$$
$$+T_{-1}(u)[2u-2z^{-1}] - 2T_{-2}(u)$$

Since $T_{-1}(u)$ and $T_{-2}(u)$ are not within the range for which (31) holds, they must both be set equal to zero. Giving:

$$F(z,u) = \frac{[2uz^{-1}-1]T_0(u) - z^{-1}T_1(u)}{[1-2uz^{-1}+z^{-2}]}$$

The starting values $T_0(u)$ and $T_1(u)$ must now be defined.

If $T_0 = 1$ and $T_1 = u$, we have:

$$F(z,u) = \frac{[2uz^{-1}-1] - z^{-1}u}{1-2uz+z^{-2}}$$

or
$$F(z,u) = \frac{z(z-u)}{z^2-2uz+1}$$

Using the 10th line in Fig. E-1 gives the solution $T_n(u) = \text{Cos}[n\,\text{Cos}^{-1}u]$ which is the Chebyshev function. $u = \text{Cos}\,\theta$. $\therefore T_n(u) = \text{Cos}\,n\theta$.

In the application of the difference equations the single value of Cos θ is used to obtain Cos $n\theta$, and with a small modification Sin $n\theta$ is also obtained.

Appendix F

EFFECTS OF QUADRATURE SIGNALS ON SERVO SYSTEMS

The usual arrangement of the Digital to Synchro converter in a control loop is shown in Chapter 4, Fig. 4-38. The signal from the control transformer is amplified and fed into a Phase sensitive detector (PSD). The DC output from the PSD forms the error signal for the control loop.

As can be seen from Fig. 4-38, the PSD is driven from the reference signal and the operation of the PSD is such as to give a gain of + 1 when the reference is positive and a gain of − 1 when the reference signal is negative. Simple consideration of the PSD will show that, if an AC voltage at the same frequency as the reference voltage but shifted in phase by 90 degrees is introduced in series with the usual input from the CT, no errors will be caused. (Errors could however be caused by the injection of such an AC voltage, if the voltage was of sufficient magnitude to cause asymmetric limiting in the amplifiers). Such an injection of a 90° phase shifted signal *is not the same* as the introduction of a real quadrature voltage caused by different phase shifts in the signal paths.

The following simple analysis shows the errors due to quadrature and how these errors are increased by a signal to reference phase shift. The signals will be considered in Resolver form to reduce the number of terms in the equations.

Referring to Fig. F-1, we will consider the reference to be in phase with the input signals and then introduce a small phase shift on one signal relative to the other.

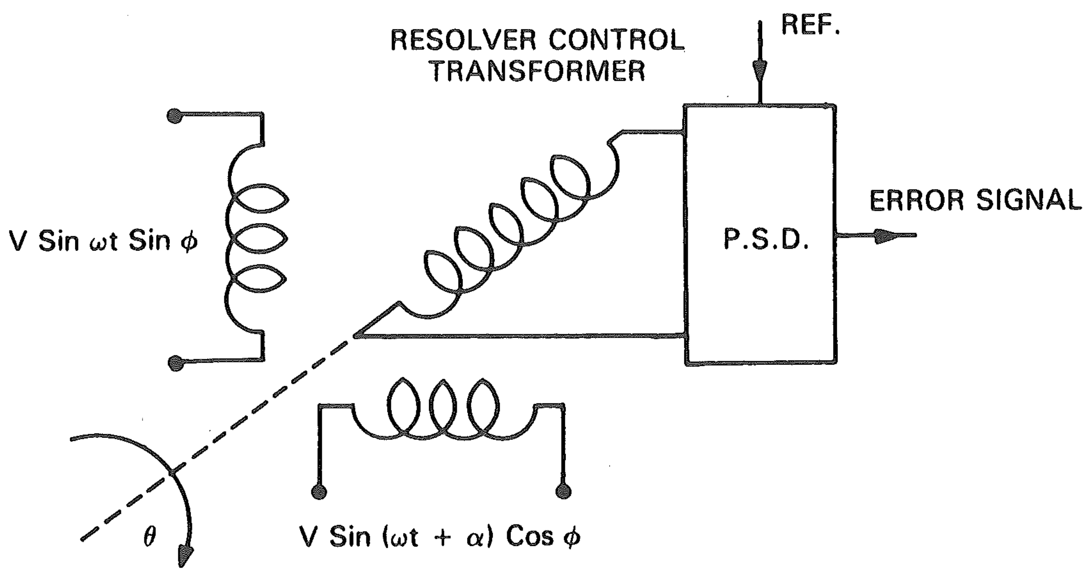

Fig. F-1 A resolver with a differential signal phase shift α.

In a perfect system the voltages would be,

$$V_s = V \sin \omega t \sin \phi$$

$$V_c = V \sin \omega t \cos \phi$$

$$V = V \sin \omega t$$

To show how the quadrature terms arises a phase shift of α will be applied to the carrier in Vc. The three voltages are now:

$$Vs = V \sin \omega t \sin \phi$$
$$Vc = V \sin (\omega t + \alpha) \cos \phi$$
$$V = V \sin \omega t$$

At a fixed angle θ the Resolver behaves like two transformers with a common secondary where the ratios are:

$$R \cos \theta \rightarrow \text{Sine input to rotor}$$
$$\text{and } -R \sin \theta \rightarrow \text{Cosine input to rotor}$$

The equivalent R for the Synchro control transformer type 11CT4b (90v L-L) is 0.64.

The voltage in the rotor winding will be the sum of the two voltages i.e.

$$V\,\text{rotor} = (V \sin \omega t \sin \phi) R \cos \theta - [V \sin (\omega t + \alpha) \cos \phi] R \sin \theta \quad \ldots\ldots(1)$$

Where ϕ is the digital angle and θ is the angle of the resolver shaft and α is the phase shift introduced into the Cosine channel.

In equation (1) the next step is to expand the term

$$\sin (\omega t + \alpha)$$

by using the relationship:

$$\sin (A + B) = \sin A \cos B + \cos A \sin B$$

Doing this we get,

$$V\,\text{rotor} = RV[\sin \omega t^{*}.\sin\phi.\cos\theta - \cos\phi.\sin\theta\,(\sin\omega t^{*}.\cos\alpha + \cos\omega t^{\dagger}.\sin\alpha)]$$

*These terms are in phase. † This is the quadrature term.

Collecting the "in-phase" and quadrature terms we have:

$$V\,\text{rotor} = RV[\sin\omega t\,(\sin\phi.\cos\theta - \cos\phi.\sin\theta.\cos\alpha) + \cos\omega t(\cos\phi\sin\theta\sin\alpha)] \quad \ldots\ldots(2)$$

A properly designed control loop with a perfect phase sensitive detector will ignore the term:

$$\cos\omega t(\cos\phi\sin\theta\sin\alpha)$$

in equation (2). For such a properly designed servo loop there is an error which is very small due to the fact that Cos α in the first term of equation (2) is not quite equal to 1.

Before proceeding to the case of the quadrature term itself we will find what the error would be in a loop which rejects the quadrature.

We make use of the fact that

$$\cos\alpha = 1 - \frac{\alpha^2}{2} + \frac{\alpha^4}{4!} \ldots\ldots\ldots (\alpha \text{ in radians})$$

and for small α (which we are considering)

$$\cos\alpha \approx 1 - \frac{\alpha^2}{2} \text{ where } \alpha \text{ is in radians.}$$

Since we are concerned for the present with a good loop which rejects the quadrature signal, we can ignore that term in the error signal. Doing this and substituting for Cos α as above gives:

$$\text{Vrotor} = \text{RV} \{ \sin \omega t [\sin \phi . \cos \theta - (1 - \frac{\alpha^2}{2}) \sin \theta . \cos \phi] \}$$

The servo will alter θ to make Vrotor = zero.

i.e. $$0 = [\sin \phi \cos \theta - \sin \theta \cos \phi + \frac{\alpha^2}{2} \sin \theta . \cos \phi] \quad .....(3)$$

In equation (3), it is the term

$$\frac{\alpha^2}{2} \sin \theta \cos \phi$$

which gives rise to the error.

For $\theta \approx \phi$, Sin θ Cos ϕ has a maximum of 0.5 at $\theta \approx \phi = 45°$

And since Sin ϕ Cos θ – Sin θ Cos ϕ $\doteqdot$ $(\phi - \theta)$ radians, the error in radians is:

$$\epsilon = \frac{\alpha^2}{2} \times 0.5 \text{ (α and error are in radians)}$$

$$\text{error in degrees} = \frac{\alpha^2 \times 0.5}{57 \times 2} \text{ (α in degrees).}$$

$$\text{error in minutes} = \frac{60 \times 0.5 \alpha^2}{57 \times 2} \text{ (α in degrees).}$$

$$\approx \frac{\alpha^2}{4} \text{ (α in degrees).}$$

i.e. ¼ arc minute for 1° of phase at worst angle of 45°

We now look at the quadrature signal which should be rejected. It is the term

RV Cos ω t [Cos ϕ Sin θ Sin α] of equation (2)

For $\phi \approx \theta$, the maximum Cos ϕ Sin θ is for $\phi \approx \theta = 45°$ and the value is 0.5.

The peak value of the quadrature is

Quadrature = α RV × 0.5 (α is in radians) (Sin $\alpha \approx \alpha$)

V = $90\sqrt{2}$

R = 0.64 for the 11CT4b Control Transformer (90 volts L-L)

and for α = 1/57 (1 degree).

The rms value of the quadrature on the rotor is $\frac{90 \times 0.64 \times 0.5}{57}$ volts per degree.

rms Quadrature Voltage on rotor = 0.5 volts per 1 degree of differential phase shift.

i.e. For 100mV rms the differential phase has to be 0.2 degrees.

The foregoing example of the effect of quadrature was based on the assumption that the signals and reference were in phase before the small differential phase shift was introduced. In practical systems there is often a phase shift of several degrees between the signals and reference. This phase shift should be minimised in the design of Synchro systems. Both control transmitter and control receiver Synchros give rise to phase leads between the input and output voltages and balancing phase leads are often introduced into the reference before it is used to drive the phase sensitive detector. DSC's generally give negligible phase shift between the input reference and output signals, if therefore a CX is replace by a DSC, the phase angle between the CT output voltage and the reference will be changed and a corresponding correction should be put in to the system at some point.