I Basic Operations

This chapter offers a brief historical and philosophical perspective of the nonlinearity scene and summarizes the principal features of useful nonlinear phenomena.

LINEARITY VS. NONLINEARITY

An ideal linear device is one for which cause and effect are proportional* for all values of inputs and output. For the great majority of analog circuits, *linear* relationships are sought. Natural devices, though, are in general nonlinear. But they are often linear enough over limited ranges to be quite useful. Much design effort is expended in finding and using devices having exceptional linearity, and in conditioning and normalizing signals to match the linear range of a device with that of the signal.

Because of the often exquisite difficulty of finding or designing devices with sufficient linearity for many tasks, the word *nonlinear* has come to have a pejorative connotation.

But devices and circuits can be designed to have nonlinear relationships that are well-defined, controllable, stable, available at low cost, and, what's more, *useful*. Examples of such relationships include multiplication, square-law, log ratios, and controlled discontinuities. Just a few applications include modulation, power mea-

*The *IEEE Standard Dictionary of Electrical and Electronic Terms* (Wiley-Interscience, 1972) defines *linearity* as "a property describing a constant ratio of incremental cause and effect" and a *linear system or element* as one for which "if y_1 is the response to x_1 and y_2 is the response to x_2, then $(y_1 + y_2)$ is the response to $(x_1 + x_2)$, and ky_1 is the response to kx_1."

surement, signal shaping, and simulating or correcting for the nonlinearity of measuring devices. Many more applications are described in these pages.

As nonlinear devices with improved characteristics (i.e., stable, repeatable fidelity to the ideal nonlinear relationship) become available, better understood, and easier to manufacture and use, their already low cost will decrease further (especially through the use of monolithic integrated circuits), and they will be more widely and readily used by electronic circuit designers. The objective of this book is to help accelerate the trend.

NONLINEAR DEVICES AND ANALOG COMPUTING

The search for devices that embody many kinds of nonlinear relationships faithfully surfaced in the heyday of the analog computer, when it was necessary, at times, to simulate multiplication, division, limits, hysteresis, calibration curves, and a host of other nonlinearities. Servomechanisms, curve tracers, and diode function generators were the most popular ways of approaching these problems. (Of these, diode function generators have shown the best chances for widespread usage and future survival, but better approaches are now available.)

As analog-computer techniques became absorbed into instrumentation and general analog-circuit design, as transistors were perfected and integrated circuits appeared, functions that had been performed by expensive and unwieldy rack-mounted packages began to become available as modular (and eventually integrated-circuit) components, starting with the operational amplifier. They soon came to include multiplier-dividers, log circuits, diode function generators, and increasingly-complex operations. Multiplication and logarithmic circuit designs could be based on inherent natural properties of transistors, operational amplifiers could be used freely to sharpen diode thresholds, and good, fast, simple comparators and electronic switches became available.

This revolution in cost, simplicity, improved performance, and the ready availability of useful nonlinear devices is quite recent. The time is ripe to stand back and take inventory of some of these riches, viewed in the perspective of their family relationships.

A LIST OF USEFUL NONLINEAR OPERATIONS

Here are a number of nonlinear operators that are useful as building blocks in circuits, apparatus, instruments, and systems. They are based on practical devices that owe their functional efficacy to one or more of a few basic properties of transistor and operational amplifier circuits.

1. *Transconductance* as a linear function of collector current (multipliers)
2. *Base-emitter forward voltage* as a logarithmic function of collector current (logarithmic devices)
3. *Presence or absence of current* as a function of polarity of the applied voltage (switches and comparators).
4. *Near-perfect temperature compensation* as a consequence of monolithic matching of devices
5. *Near-ideal transconductance and transresistance* inherent in op amp circuitry (voltage-to-current and current-to-voltage conversion)
6. *Crisp switching* as a result of high gain in op amp and comparator circuits

Nonlinear devices may be classified according to their smoothness. If the function is smooth and differentiable (except perhaps at its extremities), it may be classed as a *continuous function.* If it has one or more discontinuities or "jumps" (e.g., comparators), or if its first derivative has discontinuities (e.g., piecewise-linear functions), it is classed as a *discontinuous function.*

Basic Continuous Functional Operations

- Multiplication
- Division (ratios)
- Squaring
- Square-Rooting
- Logarithms
- Exponentials (antilogarithms)

Basic Discontinuous Functional Operations
- Ideal Diode
- Controlled Switches
- Comparators

Derived Continuous Functional Operations
- Arbitrary Exponents
- True Root-Mean-Square
- Log Ratio (two variables)
- $\sinh^{-1}$ ("AC Logarithm")
- Vector Sum
- Trigonometric Functions

Derived Discontinuous Functional Operations
- Absolute Value
- Bounds
- Dead Zone
- Jump and Window Functions
- Hysteresis

For the most part, the devices described in this chapter, in the order listed, are discussed in terms of their ideal "black box" response. That is, their inputs and outputs are in terms of voltage, and they are free from loading errors. Practical device characteristics are discussed in succeeding chapters. It is important to bear in mind that since most of these useful functions either involve operational amplifiers or are generated by transconductances, *current* may be a basic (and probably accessible) input or output.

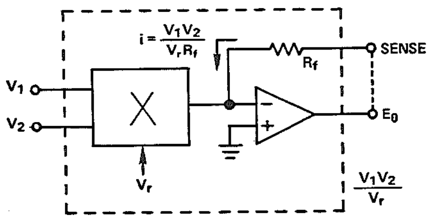
MULTIPLICATION

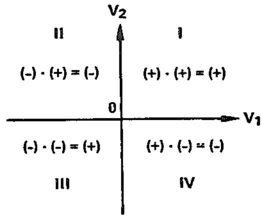
A two-input *multiplier,* in response to two input voltages, supplies their product, multiplied by a dimensional (V^{-1}) constant

$$E_0 = \frac{V_1 \cdot V_2}{V_r} \qquad (1)$$

A commonly-used range of voltages is ±10V for both inputs and the output. In this case, V_r = 10V. (Note that 10X10/10 = 10.)

If the output and both inputs can have either positive or negative polarity, and if the polarity relationships are consistent, the multiplier is called a "4-quadrant" multiplier. If response to only one of the inputs is bipolar, it is a 2-quadrant multiplier; if all signals are of a single polarity, it is a 1-quadrant device. The "quadrants" are those that would be found in the V_1-V_2 plane if one pictures the output axis as perpendicular to the plane of the paper.





Multipliers are usually furnished with an extra terminal that allows the feedback path around the output amplifier to be completed externally. In addition to facilitating gain adjustment, this terminal permits the multiplier to be used as a divider or square-rooter, as will be shown. Multipliers often have one or more differential inputs, to deal with off-ground signals.

Besides multiplication in analog computing, multipliers can be used for squaring, modulation, dynamic gain setting, and power measurement. They are available at low cost in I.C. form, and in a wide variety of performance specifications (and prices) in the form of compact discrete modules.

The scale factor, $1/V_r$, though usually fixed (with allowance for trim), is often manipulable (in some versions) by an externally-applied voltage or current, which is, in fact, a third input.

Complete multipliers involve many techniques of design. The major weight of discussion in this book is given to transconductance, logarithmic, and pulse-width-and-height-modulated types. These cover a wide range of performance capability; they are compact, low-cost, and reliable. Such other all-analog types as quarter-square, magnetic, modulated triangular-wave, servo, Hall-effect, and "slaved" multiplier designs, though historically feasible for design and widely used in specialized applications, are outside the scope of this book. Multiplying D/A converters are discussed in the *Analog-Digital Conversion Handbook**, and elsewhere in the literature. They also are outside the scope of this book, which is almost entirely devoted to purely-analog technology.

There are available on the market integrated-circuit quasi-multipliers that require a large amount of external circuitry to operate as ideal "black-box" multipliers. They, and such other specialized devices as "balanced modulator" chips, are considered only as circuit elements that embody (incompletely) the fundamental principle of transconductance multiplication.

Though it might appear at first glance that much of the technology has been thus foreclosed arbitarily, the reader will find that the practical aspects of understanding and applying multipliers covered in this book will stand him in good stead if he wishes to consider other approaches.

DIVISION AND RATIOS

A *divider* usually has two inputs. The output is the ratio of the two inputs, multiplied by a dimensional (V) constant.

$$E_0 = V_r \frac{V_2}{V_1} \qquad (2)$$

The commonly-used ranges for division are ±10V for V_2, 0^+ to +10V (or 0^- to -10V) for V_1, and ±10V for E_0. For such applications, V_r is 10V.

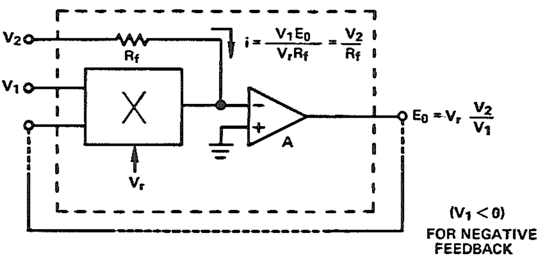
Practical dividers are either two-quadrant or 1-quadrant, depending on whether or not the numerator may be bipolar.

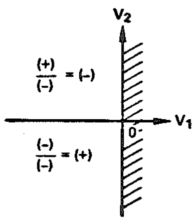
Three techniques are widely employed for division: the use of multipliers in feedback loops, multiplier designs in which the scale factor is variable, and open-loop division using logarithmic elements.

*Analog Devices, Inc., 1972, 402pp. illustrated, $3.95.

Fast, accurate multipliers, when used in feedback loops, provide fast, accurate 2-quadrant division over small dynamic ranges of denominator. This approach is useful in ratiometric measurements, where it is necessary to correct for minor variations in a measurement that are caused by variations of the reference, e.g., in strain-gage and other bridge-type measurements. The weakness of the feedback approach is that errors tend to be inversely proportional to the magnitude of the denominator; dynamic ranges greater than about 30:1 are untenable, even if high-accuracy multipliers are used.

The variable scale constant and logarithmic approaches, on the other hand, permit wide excursions of the denominator (as long as the output can be expected to remain within bounds), and a considerably closer approach to zero. Accuracy is moderate, but response tends to be slow at low levels. Variable scale-constant multipliers are two-quadrant devices; however, with log devices alone, best results are obtained in single-quadrant operation.





One should not expect an analog divider to be capable of division by zero, or by bipolar numbers. (With switching, 4-quadrant division is feasible – away from zero.) Generally, the problem can be restated to eliminate such anomalous operations. A divider can, of course, compute reciprocals if the numerator is held fixed at an appropriate constant value.

Division and multiplication are often combined in a single device (i.e., V_r is a variable input signal).

SQUARING

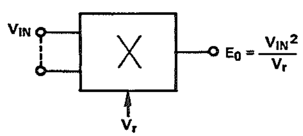
A *squarer* provides at its output a voltage proportional to the square of the input, multiplied by a dimensional (V^{-1}) constant.

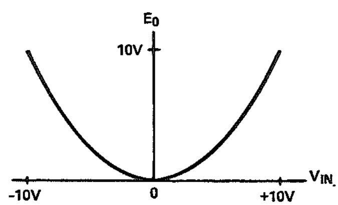
$$E_0 = \frac{V_{in}^2}{V_r} \qquad (3)$$

Typical ranges for a 2-quadrant squarer are ±10V for V_{in}, 0 to 10V for E_0, and 10V for V_r. In 1-quadrant squaring, V_{in} is of only one polarity. A 1-quadrant squarer, preceded by an absolute-value circuit ("full-wave rectifier"), will provide 2-quadrant squaring.

A squarer is useful in such operations as power measurement, wave-shaping, frequency doubling, and —used as a feedback element— square-rooting. Until simpler means of multiplication became available, one of the most important applications of squarers used to be in "quarter-square" multiplication.*

A transconductance multiplier with identical input voltages produces a wideband two-quadrant square with excellent functional fidelity at low cost. Other means of squaring include piecewise-linear diode-resistor-network approximations, semiconductor characteristics (e.g., FET's) and "dithering" of an ideal diode characteristic with a triangular wave, followed by filtering.





The "odd-function" square $x|x|$ has an output that is proportional to the square of the input but takes on the polarity of the input. It can be generated with two 1-quadrant squarers and a few op amps, or by interposing an absolute-value device between the two inputs of the multiplier used as a squarer.

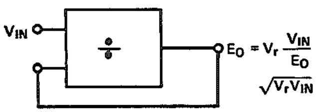
$^*x \cdot y \doteq \frac{1}{4}(x+y)^2 - \frac{1}{4}(x-y)^2$

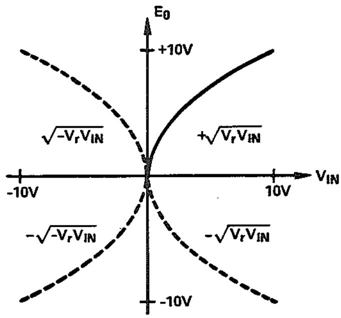
SQUARE-ROOTING

A *square-rooter* is a 1-quadrant device that computes either the positive or the negative square root of an input voltage multiplied by a dimensional (V) constant of appropriate polarity.

$$\pm E_0 = \sqrt{V_r \cdot V_{in}} \quad \text{or} \quad \pm E_0 = \sqrt{-V_r \cdot V_{in}} \qquad (4)$$

For 10V full-scale input and output, V_r is 10V. Since the slope of the square-root is theoretically infinite at zero, one might expect the largest errors to occur near zero, with slow response, and perhaps even hysteresis. Furthermore, since real square roots occur only for positive arguments, if the input signal can change sign, it may be necessary to constrain either the input or the output (or both) to prevent "lockup." Such constraints are mandatory if square-rooting is achieved by feedback around a divider that in turn involves feedback around a multiplier.





Square-roots are used in root-mean-square and vector computations, physical measurements, and simulation of fluid-flow parameters. The most popular means of square rooting are log-antilog ($\sqrt{x} = \epsilon^{\frac{1}{2}\log_\epsilon x}$) and feedback around a divider ($z = ky/z$). The log-antilog approach produces good accuracies, wide dynamic range, and benign behavior through zero. The divider approach is capable of higher speed and better accuracy near full scale and over modest dynamic ranges. Square roots computed by feeding back around $V_A V_B/V_C$ devices can use the second multiplicative input for obtaining the geometric mean ($\sqrt{V_A V_B}$).

The "odd function" square root $x/\sqrt{x}$ has the polarity of the input and an output proportional to the square-root of the input. It can be generated by an odd-function squarer in a feedback loop.

LOGARITHMIC CIRCUITS

The ideal inverting voltage-to-voltage logarithmic circuit generates the function

$$E_0 = -K \log_B \left(\frac{V_{in}}{V_r}\right) \qquad (5)$$

where V_r is the normalized unity reference, the value of V_{in} for which $E_0 = 0$. V_r is usually set arbitrarily, for either full-scale input, mid-scale output, or elsewhere. It is of appropriate polarity to make the argument positive: i.e., if V_{in} is positive, V_r is also positive; if V_{in} is negative, V_r is also negative. (The logarithm of a negative argument is not defined in terms of real values.) If the logarithmic device accepts a current input, I_{in}, the dimensional constant, V_r, is replaced by a current reference, I_r.

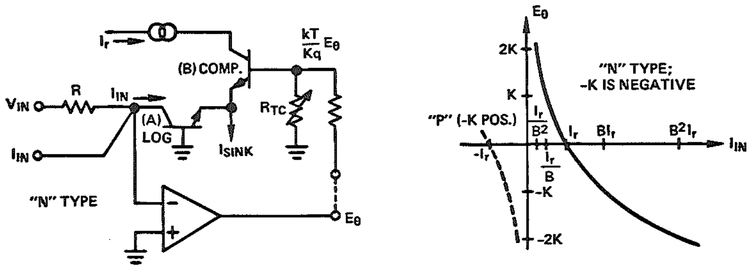
K (also a dimensional constant) is the scale factor, the number of volts corresponding to a ratio equal to the base B. For example, if B is 10, K is the number of volts corresponding to $V_{in}/V_r = 10$ (i.e., 1 decade). If B is ϵ, K is the number of volts corresponding to the ratio ϵ. If B = 2, K is the number of volts per *octave*. Popular values of K are 1 or 2 volts per *decade*. To compute the equivalent K for any other base (B'), multiply K by $\log_{10}B'$ to obtain K'; e.g., 1V/decade = 0.3010V/octave.

Logarithms are useful in signal compression, measurement of quantities having wide dynamic range, displaying information in "decibel" form, linearization of logarithmic data, computation of powers and roots, and wide-range division.

Today's logarithmic circuits are almost universally based on the relationship between the collector current and V_{BE} in a diode-connected transistor in the feedback circuit of an op amp. The transistor may be connected either as a two-terminal diode*

*A high-β transistor should be used.

(collector tied to base) or as a three-terminal element with one terminal (usually the collector, for β-independence) fixed at the summing-point potential of the op amp.



Since a diode's I_r and K both vary substantially with temperature (about 8%/°C and 0.8%/°C, referred to the current), it is important to compensate for temperature variations in practical applications. I_r variations are substantially cancelled by driving a fixed current through a matched diode (usually a twin on a monolithic chip) and taking the difference of the V_{BE}'s. K increases linearly with temperature at 1/3%/°C of its value at +27°C. In log operation, it is usually compensated for by a resistive divider having an equal temperature coefficient.

Logarithmic elements are commercially available in several forms having varying degrees of flexibility (and cost):

(1) As complete compensated current- or voltage-to voltage log modules (e.g., Model 755)

(2) As internally-compensated voltage-to-current antilogarithmic feedback elements, requiring an external operational amplifier (Model 752), and

(3) As matched transistor pairs with compensating resistive divider (Model 751), and as monolithic transistor pairs (AD818).

To ensure proper polarity relationships without excessive use of external operational amplifiers, logarithmic devices are available with a choice of polarities: "P" devices (-K positive, V_r and V_{in} negative) utilize PNP transistors to supply positive current to a summing point in response to negative inputs; "N" devices (-K negative, V_r and V_{in} positive) utilize NPN transistors to sink current at a summing point in response to positive inputs.

ANTILOG CIRCUITS

The inverse of the logarithm, the *exponential* relationship, is of the form

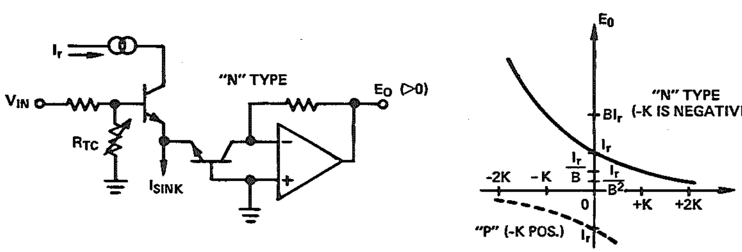
$$E_0 = V_r B^{-V_{in}/K} = V_r \log_B^{-1}\left(\frac{V_{in}}{-K}\right) \qquad (6)$$

For example, if B = 10, and K = 1V/decade,

(a) For $V_{in} = 0$, $E_0 = V_r$
(b) For $V_{in} = K$, $E_0 = 10V_r$
(c) For $V_{in} = -K$, $E_0 = V_r/10$
(d) For $V_{in} = 2K$, $E_0 = 100V_r$

Since real exponentials are always positive, E_0 is of the same polarity as V_r. In practical circuits that use inverting operational amplifiers, K and V_r are of the same polarity; -K is positive for "P" devices, and negative for "N" devices (see Logarithmic Circuits).

Antilog devices are usually employed in connection with operations on logarithmic variables. For example, if a number of input variables are to be raised to powers, multiplied and divided, they might be individually converted to logarithmic form, then summed or differenced, with weights corresponding to the individual exponents, and anti-logged.

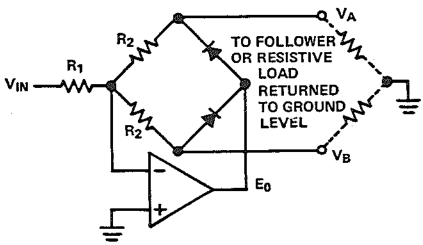


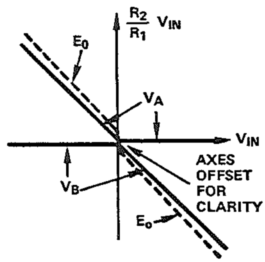
Antilog devices use the same circuitry as log devices; the distinction lies in the way they are connected. The log transistor and its temperature-compensating circuitry develop a current that is proportional to the antilogarithm of the applied voltage. If the applied voltage is the input signal, the op amp, with a feedback resistor, develops an exponential output voltage; if the applied voltage is the amplifier's *output,* it will be constrained at the value required to balance the input current (that is, the output will be proportional to the log of the ratio of I_{in} to I_r).

THE "IDEAL DIODE" OPERATOR

For switching purposes, the "ideal diode" is a one-way switch that is open when the imposed voltage is of one polarity and closed when the polarity is opposite. The *ideal diode operator* is a voltage-to-voltage circuit that would have the same response as a circuit that used an ideal diode as the switching element: the output voltage is zero for one polarity; it increases linearly with input when the polarity changes. The ideal diode operator can also be considered as a "zero-bound" circuit.

Ideal-diode operators are useful in precision dead-zone, bounds, and absolute-value circuits, and in function fitting with piecewise-linear approximations. Previously unthinkable in terms of economy, such circuits now benefit by the linearity, stability, and very low cost of IC operational amplifiers.



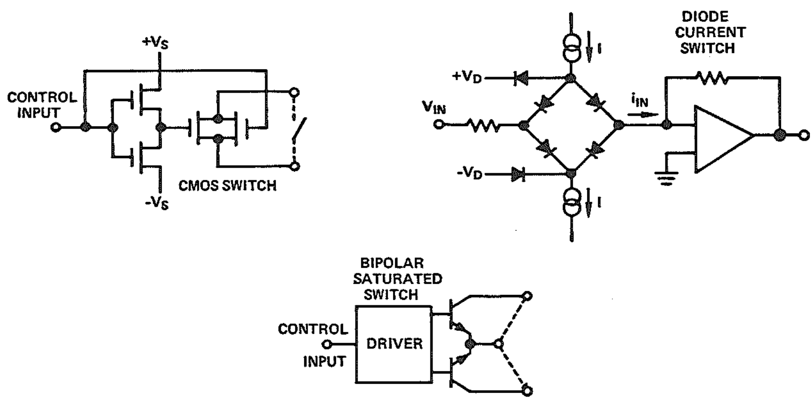


CONTROLLED SWITCHES

Though not strictly nonlinear devices (one considers switches as elements of linear time-dependent functions, controllable by outside influences), and although their characteristics and applications would require a monograph many times the size of this volume, we mention them here briefly, because they are pertinent to useful nonlinear functions.

Switches, often operated by comparators, are used locally in nonlinear circuitry to establish new conditions when thresholds have been crossed by an input or an output voltage or current. For example, they may change a gain, reverse a polarity, or initiate a new mode of operation, thus producing an overall nonlinear response. Analog switches are often an integral part of nonlinear devices, e.g., pulse-height-width multipliers, V-to-f converters. They are of course essential to multiplexers and most types of D/A and A/D converters.

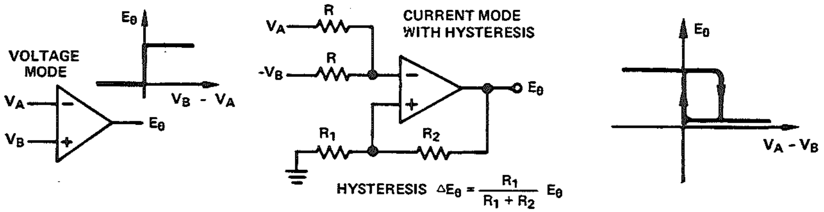
Switches come in a wide variety of forms, ranging from electromechanical, optoelectronic, and Hall-effect relays, for switching-with-isolation, to straight logic-operated electronic devices. These last are of two basic kinds: *voltage* switches and *current* switches. Examples of voltage switches, which open or close a circuit, include MOS and other FET types, as well as saturated bipolar transistors. Current switches, which switch by diverting a current, usually involve diodes or bipolar transistors, operating in the linear region. They are capable of high speeds.



COMPARATORS

Comparators are devices that have two stable output states. They signal whether an input current or voltage has crossed a threshold imposed by one or more other currents or voltages, either fixed or variable.

They are used as polarity sensors, as digital inputs to analog-controlled logic systems; they operate switches, sharpen transitions, quantize analog voltages, and –followed by switching or precision bounding– they can serve as elements of waveform generators.



A comparator is similar to an operational amplifier, in that it usually has high gain and a sensitive low-drift differential input circuit. Comparators are essentially open-loop devices; therefore internal frequency compensation against external loop closure (an important feature of op amps) is not needed. The result is higher switching speed than might be obtained with an op amp used as a comparator. Since only small amounts of negative feedback are needed for a comparator to oscillate, great pains must be taken, both in design and use, to separate the input and output circuits, electrically, physically, thermally, and at the power terminals.

Though comparators are differential devices, the common-mode range permitted with some types may be quite small. Depending on the design, the two stable comparator output levels may be compatible with standard digital logic levels (e.g., TTL), with full-range op-amp output levels (e.g., >±10V, for ±15V supplies), or with high-voltage, high-current swing capacity, for relay drive. To prevent ambiguity of the switching level due to noise, hysteresis can be effected by feeding back a small fraction of the output to the positive input terminal. In addition, some comparators can be latched in response to a digital signal.

Comparators are used either in a *voltage* mode, with two voltages to be compared applied to the two inputs, or in a *current* mode, where the voltage developed by passive summation of two or more currents is compared with a reference level, usually near "ground".

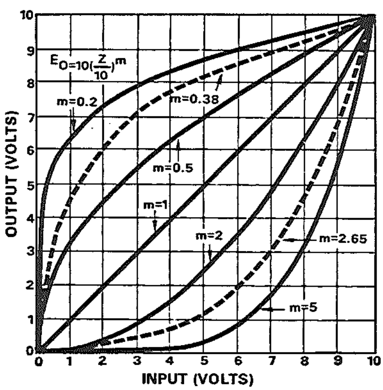
ARBITRARY EXPONENTS

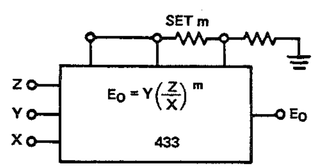
The antilog device is an exponential function. That is, it raises a constant base to a power determined by the input signal. The devices discussed in *this* section raise an input voltage ratio to an arbitrary *power,* multiplied by a dimensional (V) quantity. For three inputs, V_Y, V_Z, and V_X (all $\geqslant 0$), the output is

$$E_o = V_Y \left(\frac{V_Z}{V_X}\right)^m \qquad (7)$$

where *m* is a constant that can be arbitrarily set to a fixed value, typically in a range specified as $m_{MAX} \geqslant m \geqslant 1/m_{MAX}$. For $m > 1$, the ratio is raised to the power *m*. For $m < 1$, the $1/m$th root is obtained. To obtain negative powers, ($-m$), the roles of V_Z and V_X are simply interchanged.

Arbitrary exponents are useful in analog computing that involves exponents, in linearizing nonlinear data, and in developing power-series approximations, either with conventional integral powers or roots, or with non-integral powers (e.g., 1.211, 0.735).





The design of an arbitrary-exponent device is straightforward: The log of the ratio (V_Z/V_X) is obtained, it is multiplied by the factor *m*, then the antilog is obtained. The function can be built up with log/antilog building blocks, or it can be purchased as a single, low-cost device specifically designed to do the job (e.g., the Analog Devices' Model 433). For convenience, such a device may make available a fixed low-TC reference voltage to furnish any constant inputs that may be required. The overall device gain may be keyed to this constant, e.g., for 10-volt ranges and a predetermined* reference, V_r,

$$E_0 = \frac{10}{V_r} \cdot V_y \cdot \left(\frac{V_Z}{V_X}\right)^m \qquad (8)$$

"TRUE" ROOT-MEAN-SQUARE

An ideal root-mean-square device computes the average of the squared input over a given interval, then takes the square-root of the average, viz.,

$$E_{RMS} = \sqrt{\frac{1}{T}\int_0^T (V_{in})^2\, dt} \qquad (9)$$

True RMS is useful in evaluating AC signals (including noise), in control loops (e.g., automatic gain control), and as front-end signal conditioning for AC inputs to analog and digital panel meters and data-acquisition systems.

In practice, instead of the definite integral, a "running average" is taken, usually approximated by a first-order RC lag circuit. This approximation is valid for stationary waveforms if the filter time constant is sufficiently large, and if enough time is allowed for the output to settle.

In conventional AC-instrumentation practice, a further approximation has been employed:

$$E_{RMS} \cong 1.111 \cdot \frac{1}{T}\int_0^T |V_{in}|\, dt \qquad (10)$$

This latter approximation (the mean absolute value) is valid primarily for sine waves. It leads to gross errors if applied to noise, square waves, pulses of arbitrary duty cycle, and –of course– all waveshapes of unpredictable nature, including fluctuating DC. Though widely used, it can be greatly misleading.

*by the manufacturer

The straightforward way of computing true RMS is to perform the operations in the order indicated: Square, Average, Root. It has the disadvantages of complexity, cost, and loss of resolution because of the doubled order of dynamic range $[(100:1)^2 = 10{,}000:1]$.

A Better Way is to use a 2-quadrant $V_1 V_2/V_3$ device, together with an op amp connected as a simple low-pass filter, to solve the implicit equation

$$E_{out} = \text{Ave}\ (V_{in} \cdot V_{in}/E_{out}) \qquad (11)$$

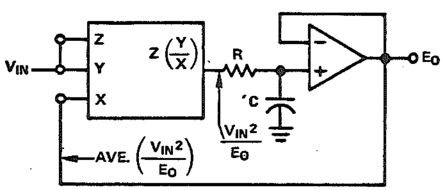
Since E_{out} can be assumed to be constant for stationary waveforms,

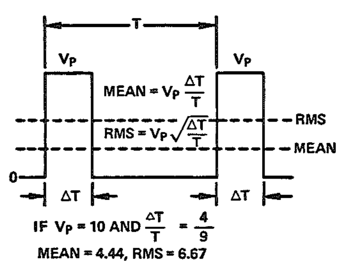
$$(E_{out})^2 = \text{Ave}(V_{in})^2 \qquad (12)$$

whence

$$E_{out} = \sqrt{\text{Ave}(V_{in})^2} \qquad (13)$$

If mean absolute value can adequately measure a known waveform (equation 10, with an appropriate multiplying factor), one can either use a full-wave rectifier with filtered output, or compute the average of $\sqrt{V_{in}^2}$ by using the *input* of the filter (i.e., the output of the multiplier) as the denominator in equation 11. Though this latter approach might seem a bit roundabout, it may be useful for apparatus requiring a choice of RMS or MAV.





LOG RATIO

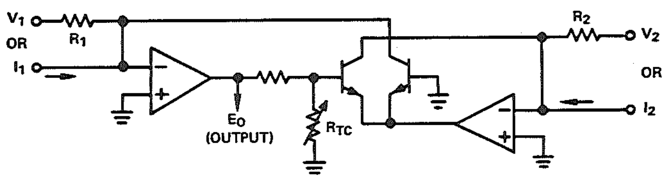
Log ratio devices can measure ratios of either voltages or currents

$$E_0 = K \log_B \frac{V_1}{V_2} \quad \text{or } K \log_B \frac{I_1}{I_2} \quad \text{or } K \log_B \frac{V_1/R}{I_2} \qquad (14)$$

They are useful where measurements involve exponential data (e.g., light measurements), where the inputs or the ratio itself can have a wide dynamic range, for gain measurements displayed in log form (e.g., "dB"), for generation of powers and roots, and for signal compression.

Log ratio devices are available as complete entities, e.g., Analog Devices' Model 756. They can also be assembled from log amplifiers, or with tne more-elementary log devices mentioned in the section on Logarithmic Circuits.

Since real values of the logarithm do not exist for negative arguments, logarithmic functions* of bipolar signals (e.g., AC signals) must be obtained in terms of properties of the signal rather than the signal itself. Examples of such properties include mean-absolute, RMS, and peak measurements.



SINH^{-1} OR "AC LOG"

The inverse hyperbolic sine characterizes the output of an op amp that has two complementary antilog transconductors (e.g., 752) paralleled in its feedback path.

*Log ratio is computed by subtracting logarithms. However, even if the ratio were obtained first, the rules regarding zero and bipolar denominators must be observed.

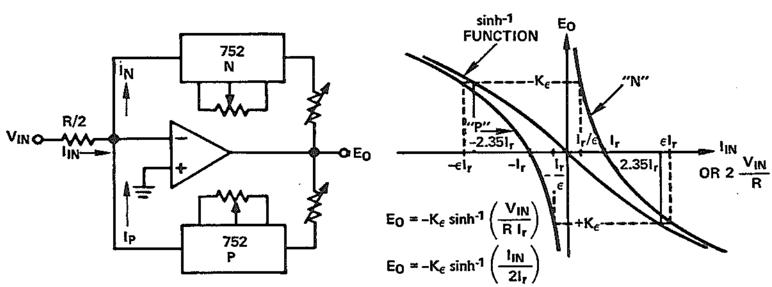
$$-2\,\frac{V_{in}}{R} = I_r\,\exp(E_0/\text{-}K) - I_r\,\exp(E_0/K) = 2I_r\,\sinh(E_0/\text{-}K) \qquad (15)$$

whence

$$E_0 = K\,\sinh^{-1}\left(\frac{V_{in}}{RI_r}\right) \qquad (16)$$

This useful device has logarithmic behavior over wide ranges of $+V_{in}$ and $-V_{in}$, and well-behaved linear behavior through zero. It will compress bipolar signals logarithmically in a symmetrical and predictable manner.

Graded null meter and wide-range bipolar analog panel meter are two DC applications; non-saturating signal compression is a typical AC application. With AC signals, the choice of I_r is compromised by the conflicting demands of bandwidth and dynamic range.



If the antilog elements are connected in the input path, the inverse function, i.e., the hyperbolic sine, is generated.

In the simplest (but least stable) form of this device, the antilog elements may be a pair of diodes connected in parallel, back-to-back.

VECTOR SUM (MAGNITUDE)

A vector-sum device computes the square-root of the sum of the squares of the inputs

$$E_0 = \sqrt{V_1^{\,2} + V_2^{\,2} + \ldots + V_n^{\,2}} \qquad (17)$$

Typical applications include summation of orthogonal measurements, such as length, force, or voltage vectors, and measures of random statistical quantities. Geometrical quantities, such as sums of areas, and diagonals of rectangular n-dimensional figures are also obtainable.

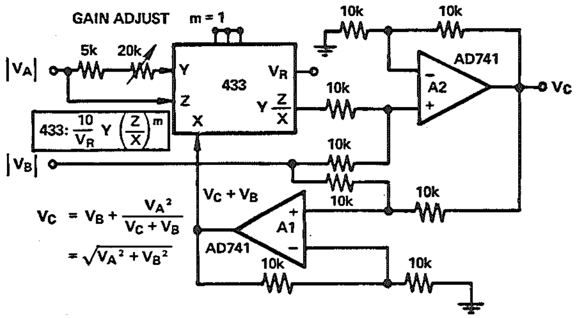
There are two popular approaches to computing vector sums: direct and implicit. The *direct* approach requires that each input be individually squared, the sum be taken, and the result square-rooted. While the method is straightforward, the expansion of dynamic range inherent in squaring imposes serious limitations on accuracy if E_0 is to vary over a wide dynamic range. For *n* variables, *n* squarers, a square-rooter, and a summing amplifier are required.

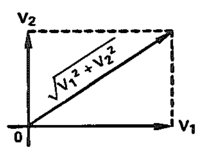
The *implicit* approach, which is capable of yielding more-accurate results, calls for implementation of the equation

$$E_0 = \frac{V_1^2}{E_0 + V_n} + \frac{V_2^2}{E_0 + V_n} + \ldots + \frac{V_{n-1}^2}{E_0 + V_n} + V_n \quad (18)$$

It requires n-1 devices that compute $V_a \cdot V_b / V_c$, and two summing amplifiers.

The special (and most-frequently encountered) case, in which n = 2, requires a single $V_a \cdot V_b / V_c$ device and 2 modest-performance op amps. This compares favorably in cost, complexity, and performance with the direct approach, using 2 squarers, one rooter, and an op amp.





TRIGONOMETRIC FUNCTIONS

Useful trigonometric functions include A $\sin\theta$, A $\cos\theta$, r $\sin\theta$, r $\cos\theta$, their combinations in sums and products, and $\tan^{-1}(V_y/V_x)$.

They are applied in vector resolution and composition, coordinate transformations, waveshaping, and function generation.

Trigonometric relationships among analog variables can be simulated roughly by simple circuits involving FET or transistor characteristics, and to greater accuracy by piecewise-linear diode function fitting or by power-series approximations.

A recent significant development in power-series approximations is the use of non-integral powers, computed by adjustable-exponent devices, such as the Model 433. This approach can significantly decrease the number of power terms required for a given level of accuracy. For example, $\sin\theta$ can be approximated by $(x - x^3/6.79)$ to within 1.35% over the range 0 to $\pi/2$; but the approximation $(x - x^{2.827}/6.28)$ has less than 0.25% error over the same range of angle.

In analog-digital function fitting, trigonometric relationships may be stored in read-only memories (ROM's) and returned to analog form by D/A conversion.[1]

The most important consideration (other than the basic fit) affecting cost and complexity of designs involving trigonometric functions is the range of angle. Since both the input and the output of a "$\sin\theta$" device are usually voltages, the function that is really being fit is (for example) $V_{FS} \sin\frac{\pi}{2}(V_{in}/V_r)$. V_{FS} is the voltage cor-

TYPICAL APPROXIMATIONS:

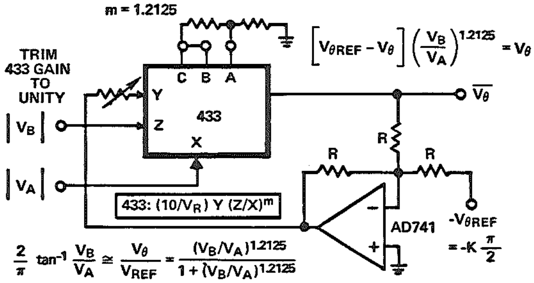
$$\sin\theta \cong \theta - \theta^{2.827}/6.28$$

$$\cos\theta \cong \left[\frac{\pi}{2} - \theta\right] - \frac{1}{6.28}\left[\frac{\pi}{2} - \theta\right]^{2.827}$$

$$\tan^{-1}\frac{V_B}{V_A} = \theta \cong \frac{\pi}{2}\,\frac{(V_B/V_A)^{1.2125}}{1 + (V_B/V_A)^{1.2125}}$$

[1] See *Analog-Digital Conversion Handbook*, Analog Devices, 1972, page I-65 *et seq.*

responding to the sine of 90°, and V_r is the voltage corresponding to 90°.

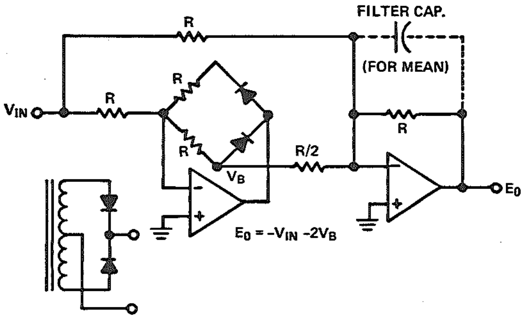


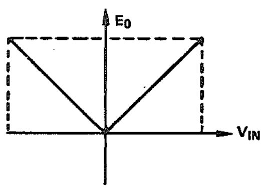
The simplest designs are those involving a single quadrant, $0° \leqslant V_{in} \leqslant V_r$. Those involving two quadrants, e.g., $-V_r \leqslant V_{in} \leqslant V_r$, are not too much more difficult (the linear term alone is within 0.25% to -14°). But if many quadrants are required, and especially if the angle can increase without limit, some form of switching and polarity sensing is necessary to continually translate the function to the first (two) quadrant(s), maintaining appropriate polarity relationships.

ABSOLUTE VALUE

An absolute-value device, otherwise known as a "full-wave rectifier," measures the instantaneous magnitude of the departure of a voltage from zero. The output may be assigned an arbitrary positive or negative polarity, depending on the circuit application.

$$E_0 = \pm |V_{in}| \tag{19}$$





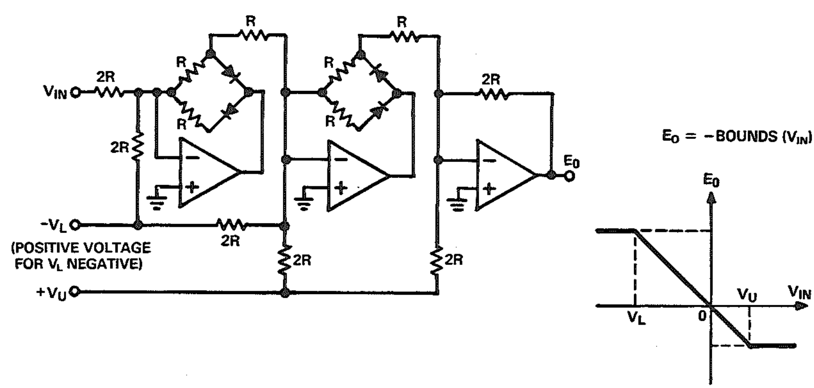
Applications in precision instrumentation include AC measurements, function fitting, inputs to single-quadrant devices (squarers or vector sums), triangular-wave frequency doubling, and error measurements.

The best-known embodiment of absolute value is the conventional "full-wave rectifier" circuit, in which two diodes, sharing a common terminal, are driven out-of-phase by $+V_{in}$ and $-V_{in}$. Depending on diode polarity, the output will always be either plus or minus the magnitude of the input, less 1 diode drop. In precision instrumentation, circuits involving operational amplifiers are used (in a number of possible configurations) to eliminate the diode drop (and its variations with current and temperature).

BOUNDS

A bounding circuit has an output that is linear for inputs up to a preset value, and unchanging beyond it. *Upper* and *lower* bounds are often used together; either may be fixed or variable, depending on the application. For an input, V_{in}, and upper and lower bounds V_U and V_L,

$$\begin{aligned} E_0 &= V_{in} && (V_L \leqslant V_{in} \leqslant V_U) \\ E_0 &= V_U && (V_{in} \geqslant V_U) \qquad (20) \\ E_0 &= V_L && (V_{in} \leqslant V_L) \end{aligned}$$



Precision bounds are useful in setting, or simulating, limits to voltage or current (or its rate-of-change), in establishing thresholds of ranges of operation in piecewise-linear function fitting, and (preceded by comparators) in establishing precise voltage staircase functions (i.e., quanta).

Bounds can be implemented with diodes and/or transistors, employing matched pairs to provide first-order threshold cancellation. However, greater accuracy can be obtained by using diodes with operational amplifiers to form "ideal diode" circuits, in which the diodes serve only as switches, with their thresholds corrected-for by the inherent properties of high-gain operational-amplifier loops.

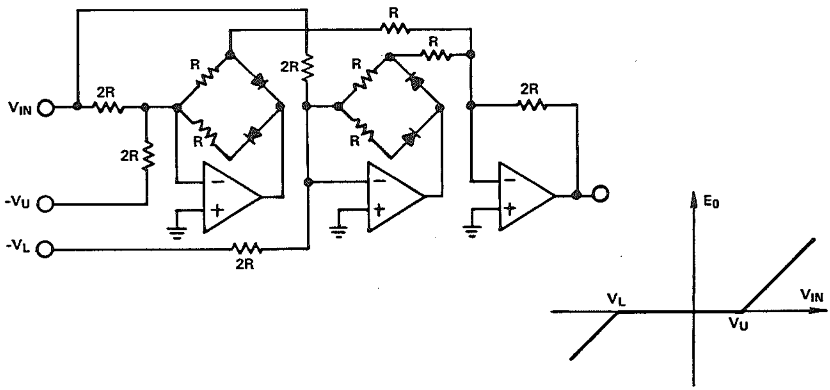
DEAD ZONE

In a dead-zone operation, the output is typically a linear function of the input, except for a band that is insensitive to the input. That is, for an input V_{in},

$$\begin{aligned} E_0 &= 0 & V_L \leqslant V_{in} \leqslant V_U \\ E_0 &= V_{in} - V_U & V_{in} \geqslant V_U \\ E_0 &= V_{in} - V_L & V_{in} \leqslant V_L \end{aligned} \qquad (21)$$

Dead-zone is related to bounds

$$E_0\,(Z) = V_{in} - E_0\,(B) \qquad (22)$$



Like bounds, dead-zone is useful in setting thresholds for piecewise-linear function generation. It is also useful in suppressing noise in the vicinity of a null, in generating "linear" hysteresis functions, and in stabilizing the amplitude of "limit cycle" oscillations. It has also been used for velocity modulation of oscilloscope intensity, and –with dither– for fitting parabolic functions.

Dead zone, like bounds, can be established simply but not very accurately with diodes and transistors, or, –with great accuracy, at low frequencies– by ideal-diode circuits.

JUMP AND WINDOW FUNCTIONS

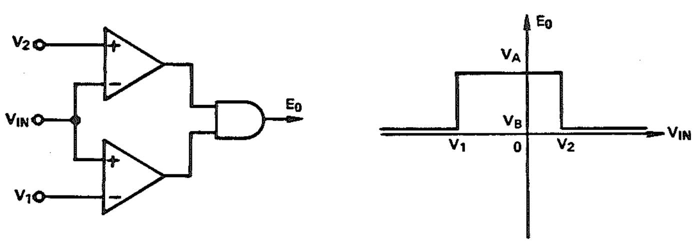
A "jump" function is simply the output of a comparator

$$\begin{aligned} E_0 &= V_A \qquad (V^+ - V^-) > 0 \\ E_0 &= V_B \qquad (V^+ - V^-) < 0 \end{aligned} \tag{23}$$

The outputs of two TTL-compatible comparators "hard-wired" together can produce a "window" in the response to a common input, a band for which the output is at one level, with the response everywhere else at the second level.

$$\begin{aligned} E_0 &= V_A \qquad V_1 < V_{in} < V_2 \\ E_0 &= V_B \qquad V_2 < V_{in} < V_1 \end{aligned} \tag{24}$$

Although the comparator output levels are dependent on loading and temperature, they can be shifted and rendered quite accurate and stable by the use of precision bounds.

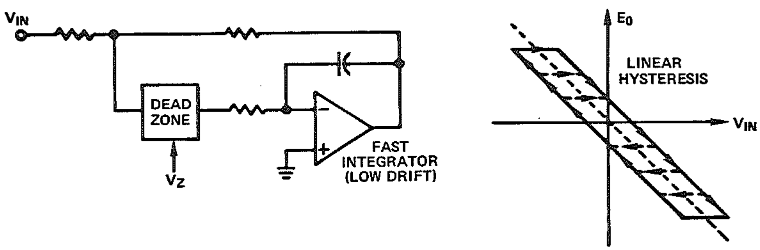


Jump functions are useful in sensing polarity, in precisely locating thresholds, and for all of the applications mentioned under Comparators. The window function can be used for precise quantization, for grading and sorting, and for pulse-width modulation. A comparator and a window can be used in conjunction to initiate a function that depends on the sign and quantized magnitude of a voltage.

HYSTERESIS

A device that exhibits hysteresis has an output that is a two-valued function of the input, over a portion of the input range. That is, the response is not uniquely dependent on the value of the input; it depends also on its history. A plot of output vs. input shows the characteristic "loop" behavior.

A familiar form of hysteresis is seen in a two-level comparator with fractional positive feedback. Once the output has switched, the input must be backed off to a second threshold beyond the switching point to cause the device to switch back to the original state. Flip flops and "latchup" phenomena rely on large amounts of hysteresis.



Besides two-level hysteresis, there is also "linear" hysteresis, best-known to electrical engineers in the form of magnetic "remanence," and to mechanical engineers as "backlash" in gear trains. Devices with linear hysteresis follow the input in one direction; when the direction of input is reversed, a dead band must be traversed before the output follows.

This form of hysteresis can be simulated by closing a feedback loop around an integrator, preceded by a dead zone. As the input increases, the integrator will follow (first-order lag), with the dead-zone output at the forward threshold. When the input is reversed, it must traverse the dead band (the integrator *holding*) before the output can be made to reverse direction.

CONCLUSION

We have seen here a wide variety of useful nonlinear phenomena, based on fundamental building blocks. In Part Two, we shall see ways in which these phenomena are, can be, and perhaps *should* be employed in solving an even greater variety of real analog design problems.