Converting between RMS and Peak-to-Peak Jitter at a Specified BER

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1 Introduction

There are several ways to quantitatively state the amount of random jitter within a system. The following discussion addresses the differences between two conventions. The first method is to give a standard deviation of the jitter distribution (or equivalently the RMS value), and the second method is to select a bit error rate (BER) threshold and define the random jitter as a peak-to-peak value.

1.1 Voltage Noise vs. Time Noise

Jitter is essentially variation in the zero crossing times of the data eye. There are two ways that noise can cause bit errors in a system. One way occurs when the noise causes the data waveform to dip below the decision threshold voltage at the sampling instance (voltage noise). Also noise can cause errors by inducing jitter (timing noise). Jitter causes errors in a system by moving the data eye across the vertical sampling instance.

1.2 RMS to Peak-to-Peak Jitter Conversion

To convert between RMS and peak-to-peak random jitter, *the BER must be specified*. The following equations can be used to convert between the two:

$$Jitter_{P-P} = \alpha * Jitter_{RMS}$$

and

$$Jitter_{RMS} = \frac{Jitter_{P-P}}{\alpha}$$

Where α is determined by $\frac{1}{2}\mathrm{erfc}\left(\frac{\alpha}{2*\sqrt{2}}\right) = \mathrm{BER}$.

Table 1. Scaling Factors (α) for Different BER Tolerances

BER	α
10^{-3}	6.180
10^{-4}	7.438
10^{-5}	8.530
10^{-6}	9.507
10^{-7}	10.399
10^{-8}	11.224
10^{-9}	11.996
10^{-10}	12.723
10^{-11}	13.412
10^{-12}	14.069
10^{-13}	14.698
10^{-14}	15.301
10^{-15}	15.883
10^{-16}	16.444

2 How Peak-to-Peak Jitter Relates to RMS Jitter

A Gaussian distribution describes random jitter. Qualitative analysis shows that the tails of a Gaussian distribution extend indefinitely on either side of the mean. Therefore, it is impossible to specify a peak-to-peak jitter range that bounds the jitter 100% of the time. Instead we want to identify a

range that contains the jitter, for example, 99.99999% of the time. This means that 0.00001% of the time the jitter will be outside of our peak-to-peak range. Calculating peak-to-peak jitter is important for jitter budget analysis. It is assumed that any samples that fall outside the peak-to-peak range will cause errors. Therefore, if a BER target of 10^{-12} is selected, it is necessary to select a range that will contain the jitter all but 0.0000000001% of the time.

2.1 Peak-to-Peak Jitter and Oscilloscope Measurements

When using an oscilloscope in histogram mode to measure random jitter, usually the measured peak-to-peak jitter is of little practical value. Most oscilloscopes generate the peak-to-peak value by simply finding the time difference between the two furthest points captured in the histogram. Because this measurement is dependant upon a number of factors, including the number of samples acquired, it is not a statistically valid figure of merit for specifying peak-to-peak jitter.

3 More about Gaussian Statistics

The assumption made in quantifying random jitter is that it is approximated by a Gaussian distribution. Generally this is true when the dominant source of noise in the system is thermal noise. In practice this seems to be valid. On the other hand, deterministic or pattern-dependent jitter is decidedly not Gaussian in nature.

Statistics textbooks tell us a Gaussian distribution can be completely defined by two parameters: mean and standard deviation. The mean of the distribution determines the horizontal location and is dependent upon the frame of reference selected. In this discussion, the ideal edge of the data eye is set to be t = 0. Figure 1 is a graph of Gaussian probability density function (PDF). The PDF is a representation of the probability of an event happening at a certain time. In this example, it shows how the zero crossing of the data will move relative to the ideal position (set at 0 on the x-axis). A fundamental property of the PDF is that the area it contains (or its integral) is equal to 1.

Figure 1 also shows how the Gaussian distribution can be limited to include a percentage (<100%) of samples. The area under the tails represents the region where the data edge has shifted beyond the bounding limits.

Assuming that samples occurring outside the bounding limits will cause errors, then once the crossing moves across the sampling instance, the probability of error is 50%, assuming uncorrelated data with 50% transition density. The probability of error (P_e) can be calculated as:

$$P_e = P(\text{crossing} < t_{sample}) * 50\% + P(\text{crossing} > t_{sample}) * 50\%$$

Because the Gaussian distribution is symmetric across t = 0:

$$P(\text{crossing} < t_{sample}) = P(\text{crossing} > t_{sample})$$

We can simplify P_e to:

$$P_e = P(\text{crossing} < t_{sample}) * 50\% + P(\text{crossing} > t_{sample}) * 50\%$$
$$= P(\text{crossing} < t_{sample}) = P(\text{crossing} > t_{sample})$$

The probability density function is described by:

$$f(t) = \frac{1}{\sqrt{2\pi}\sigma} e^{\frac{-(t-m)^2}{2\sigma^2}}$$

where σ is the standard deviation (RMS value) of the jitter and m is the mean.

If we let m = 0, we can find the area of the shaded region by integrating:

$$P_e = \frac{1}{\sqrt{2\pi}\sigma} \int_{t_{sample}}^{\infty} e^{\frac{-t^2}{2\sigma^2}}$$
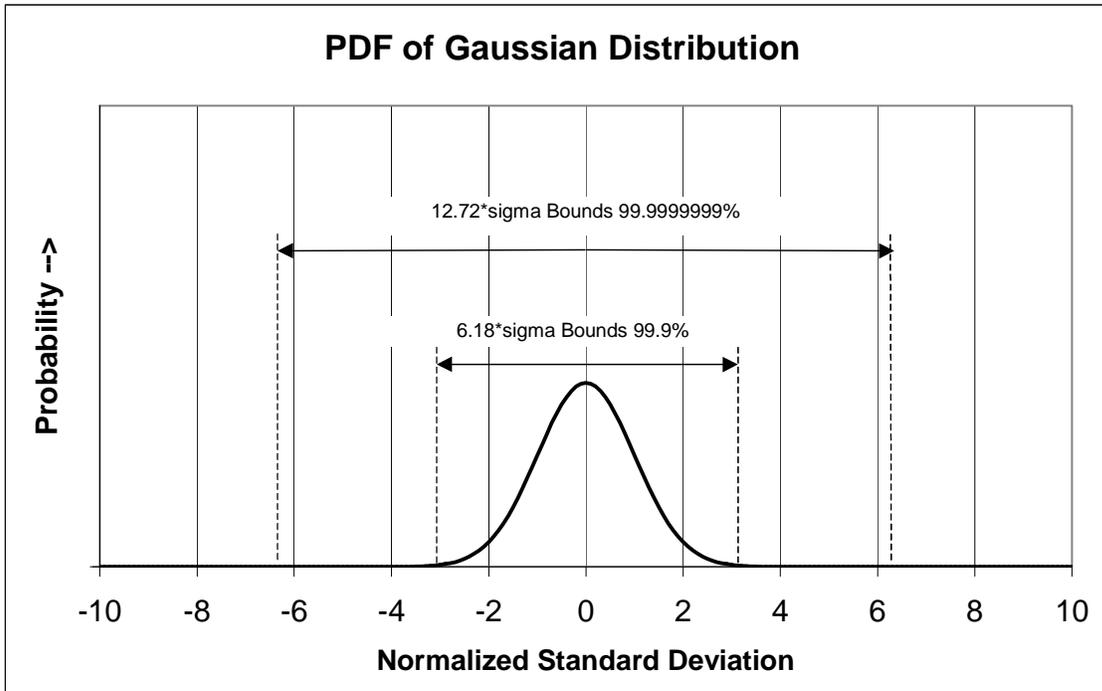
PDF of Gaussian Distribution

Figure 1. PDF of a Gaussian distribution with a variance of 1

Because no closed form solution exists for the above integral, there is a need to simplify further. Math tables showing solutions to the above equation are common for the case where $\sigma = 1$. We can normalize to $\sigma = 1$ by making a change of variables:

$$\alpha = {t_{sample}}\Big/{\sigma} = \frac{Jitter_{P-P}}{Jitter_{RMS}}$$

With this change of variables, the integration is now in the form commonly called the complementary error function (erfc). Now if the standard deviation (or RMS value) of the noise is known, the BER for different sampling times can be found using the complementary error function:

$$\frac{1}{2}\operatorname{erfc}\left(\frac{\alpha}{2*\sqrt{2}}\right) = BER$$

The erfc function is tabulated in many math references and as a function in Excel and MATLAB.

As an example, the peak-to-peak jitter can be found at a BER $= 10^{-10}$ when the RMS jitter is 5ps. Referring to an erfc table, we find that $10^{-10} = \frac{1}{2}*\operatorname{erfc}\left(12.723/(2*\sqrt{2})\right)$. The example shows that:

$$Jitter_{P-P} = \alpha * Jitter_{RMS}$$

The answer is $Jitter_{P-P} = 12.723*5ps = 63.61ps$.

Reference: Leon W. Couch II, *Digital and Analog Communication Systems,* 5th Edition, Prentice Hall 1997