Application Note:

HFAN-2.2.0

Rev.1; 04/08

Extinction Ratio and Power Penalty

maxim integrated™

Extinction Ratio and Power Penalty

1 Background

Extinction ratio is an important parameter included in the specifications of most fiber-optic transceivers. The purpose of this application note is to show how the optical extinction ratio is defined and to demonstrate how variations in extinction ratio affect the performance of digital optical communication systems.

2 Definitions

In general, digital optical communication systems transmit binary data using two levels of optical power, where the higher power level represents a binary 1 and the lower power level represents a binary 0. These two power levels can be represented as P_1 and P_0, where $P_1 > P_0$ and the units of power are watts.

In optical transmitters, electrical current is converted to optical power (electrical to optical conversion, or E/O), and, in optical receivers, optical power is converted back to electrical current (optical to electrical conversion, or O/E). The electrical currents I_0 and I_1 are proportional to the corresponding optical power levels. The responsivity, ρ, of the receiver is the constant of proportionality between the received optical power level and the current with units of amperes per watt. The responsivity can be used to demonstrate the following:

$$I_0 = \rho P_0 \text{ and } I_1 = \rho P_1 \tag{1}$$

$$I_1 - I_0 = \rho(P_1 - P_0) \tag{2}$$

$$r_e = \frac{I_1}{I_0} = \frac{P_1}{P_0} \tag{3}$$

The ratio between the "one" level and the "zero" level, shown in Equation 3, is defined as the "extinction ratio," and is represented by the symbol r_e. (Note that extinction ratio is defined in some literature as the reciprocal of Equation 3.)

In an ideal transmitter, P_0 would be zero and thus r_e would be infinite. In most practical optical transmitters, however, the laser must be biased so that P_0 is in the vicinity of the laser threshold, meaning that a finite amount of optical power is emitted at the low level and thus, $P_0 > 0$.

In many cases it is useful to talk in terms of the average optical power, P_{AVG}, and in these instances the following relationships are useful:

$$P_{AVG} = \frac{P_0 + P_1}{2} \tag{4}$$

$$P_0 = 2P_{AVG}\left(\frac{1}{r_e + 1}\right) \tag{5}$$

$$P_1 = 2P_{AVG}\left(\frac{r_e}{r_e + 1}\right) \tag{6}$$

3 Error Probability in Digital Communication Systems

The receiver in a digital communication system must make two decisions: (1) when to sample the received data and (2) whether the sampled value represents a binary 1 or 0. In order to understand the effects of extinction ratio on system performance, the following analysis focuses on the second decision.

3.1 Decisions about the Received Signal

The decision circuit in a basic receiver simply compares the sampled voltage, v(t), to a reference value, γ, called the "decision threshold." If v(t) is greater than γ, then it decides that a binary 1 was sent; whereas if v(t) is less than γ, then a binary 0 must have been sent. A major obstacle to making the correct decision is noise in the received data.

If we assume that additive white Gaussian noise (AWGN) is the dominant cause of erroneous decisions, then we can calculate the statistical probability of making an incorrect decision. The probability density function for v(t) with AWGN can be written mathematically as:

$$PROB[v(t)] = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\left(\frac{v(t)-v_S}{\sigma}\right)^2} \qquad (7)$$

where v_S is the voltage sent by the transmitter (the mean value of the density function), v(t) is the sampled voltage value in the receiver at time t, and σ is the standard deviation of the noise. Equation 7 is illustrated in Figure 1.

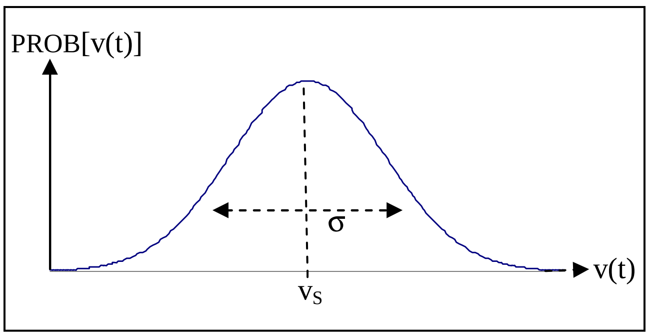

Figure 1. AWGN probability density function

In binary signaling, v_S can take on one of two voltage levels, which we will call v_{S0} and v_{S1}, and the probability of making an erroneous decision in the receiver is as follows:

$$P[\varepsilon] = P[v(t) > \gamma \mid v_S = v_{S0}]\, P[v_{S0}] + P[v(t) < \gamma \mid v_S = v_{S1}]\, P[v_{S1}] \qquad (8)$$

where $P[\varepsilon]$ represents the probability of error and $P[x \mid y]$ represents the probability of x given y. If we assume an equal probability of sending v_{S0} versus v_{S1} (50% mark density), then $P[v_{S0}] = P[v_{S1}] = 0.5$. Also, in order to simplify the example at this point, we will assume that the same noise affects either voltage level (i.e., $\sigma_0 = \sigma_1$), which means that $P[v(t) > \gamma \mid v_S = v_{S0}] = P[v(t) < \gamma \mid v_S = v_{S1}]$. Using these assumptions, Equation 8 can be reduced to:

$$\begin{aligned} P[\varepsilon] &= P[v(t) > \gamma \mid v_S = v_{S0}] \\ &= P[v(t) < \gamma \mid v_S = v_{S1}] \\ &= \int_{\gamma}^{\infty} PROB[v(t)]\, dt \qquad (9) \end{aligned}$$

where *PROB*[v(t)] is defined in Equation 7. This result is illustrated in Figure 2.

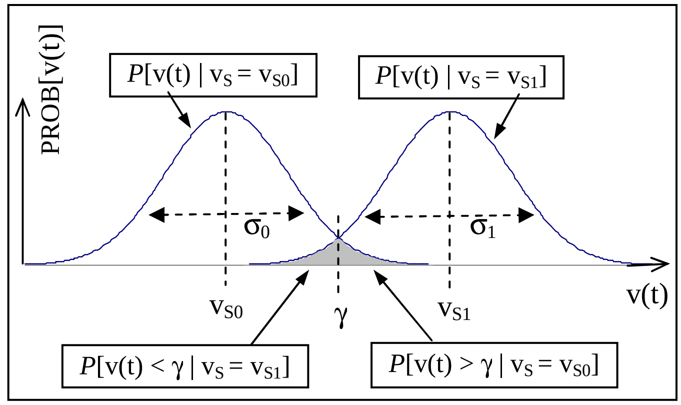

Figure 2. Probability of error for binary signaling

From Figure 2 and Equations 8 and 9, we can see that the probability of error is equal to the area under the tails of the density functions that extend beyond the threshold, γ. This area and thus the bit-error ratio (BER) are determined by two factors: (1) the standard deviations of the noise (σ_0 and σ_1) and (2) the distance between v_{S0} and v_{S1}. (Note: This is based on the assumption that $\gamma = (v_{s1},-v_{s0})/2$, which is valid only for $\sigma_0 = \sigma_1$.)

3.2 The Q Factor

The conclusion of the discussion above is that the BER is determined by the standard deviation (RMS average) of the noise and the distance between the signal levels. This conclusion can be rephrased in terms of the optical signal-to-noise ratio at the input to the optical receiver, which is defined (in units of current) as the peak-to-peak signal current divided by the RMS noise current[1]. (Note: The optical receiver converts optical power to electrical current. This current is later converted to a voltage before it is applied to the receiver decision circuit.) This form of the signal-to-noise ratio has been given the name "Q" or "Q-factor" in the literature[2] and can be written mathematically as:

$$Q = \frac{I_1 - I_0}{\sigma_0 + \sigma_1} \qquad (10)$$

The result in the denominator of Equation 10 is based on the assumption that σ_0 and σ_1 are uncorrelated (i.e., $\sigma_0\sigma_1 = 0$), so that $\sigma_0 + \sigma_1 = \sqrt{(\sigma_0 + \sigma_1)^2} = \sqrt{\sigma_0^2 + 2\sigma_0\sigma_1 + \sigma_1^2} = \sqrt{\sigma_0^2 + \sigma_1^2}$.

The BER can be written mathematically in terms of Q as follows:

$$BER = \tfrac{1}{2} erfc\left(\frac{Q}{\sqrt{2}}\right) \qquad (11)$$

where *erfc( )* is the complementary error function, defined as[3]:

$$erfc(x) = \tfrac{2}{\sqrt{\pi}} \int_x^{\infty} e^{-y^2} dy \qquad (12)$$

4 The Power Penalty

As noted in Section 2, P_0 is ideally equal to zero, making the optimum extinction ratio infinite. When the extinction ratio is not optimum, however, the transmitted power must be increased in order to maintain the same BER. This increase in transmitted power due to non-ideal values of extinction ratio is called the "power penalty."

In order to derive a mathematical expression for the power penalty, we can start by noting that, from Equations 10 and 11, if the Q-factor is held constant, then the BER will remain constant. In order to simplify the analysis, we will assume that thermal noise is dominant and equal for both the P_0 and P_1 power levels, and thus $\sigma_0 = \sigma_1 = \sigma_T$, and $\sigma_0 + \sigma_1 = 2\sigma_T$ (where σ_T is the standard deviation of the thermal noise).

Using the following algebra, we can write an expression for the Q-factor in terms of the extinction ratio, r_e:

$$Q = \frac{I_1 - I_0}{\sigma_0 + \sigma_1} \qquad \text{(Equation 10)}$$

$$= \frac{\rho(P_1 - P_0)}{2\sigma_T} \qquad \text{(using Equation 2)}$$

$$= \frac{\rho\left[\left(2P_{AVG}\frac{r_e}{r_e+1}\right) - \left(2P_{AVG}\frac{1}{r_e+1}\right)\right]}{2\sigma_T}$$

(using Equations 5 and 6)

$$Q = \frac{\rho P_{AVG}}{\sigma_T}\left(\frac{r_e - 1}{r_e + 1}\right) \qquad (13)$$

Next, we can rearrange Equation 13 in order to express P_{AVG} as a function of r_e:

$$P_{AVG}(r_e) = \frac{Q\sigma_T}{\rho}\left(\frac{r_e + 1}{r_e - 1}\right) \qquad (14)$$

From Equation 14, we can make the following important observation: *As the extinction ratio is degraded below its ideal value of infinity, the average power must be increased in order to maintain a constant value of Q and hence a constant BER.*

The power penalty is defined as the ratio of the average power required for a given value of r_e to the average power required for the ideal case of $r_e = \infty$. This can be defined mathematically as:

$$\delta_e(r_e) = \frac{P_{AVG}(r_e)}{P_{AVG}(r_e = \infty)} = \frac{(r_e + 1)}{(r_e - 1)} \qquad (15)$$

where δ_e is the power penalty and $P_{AVG}(r_e)$ is defined in Equation 14.

The power penalty as a function of extinction ratio is graphed and tabulated below for both linear and logarithmic (dB) ratios.

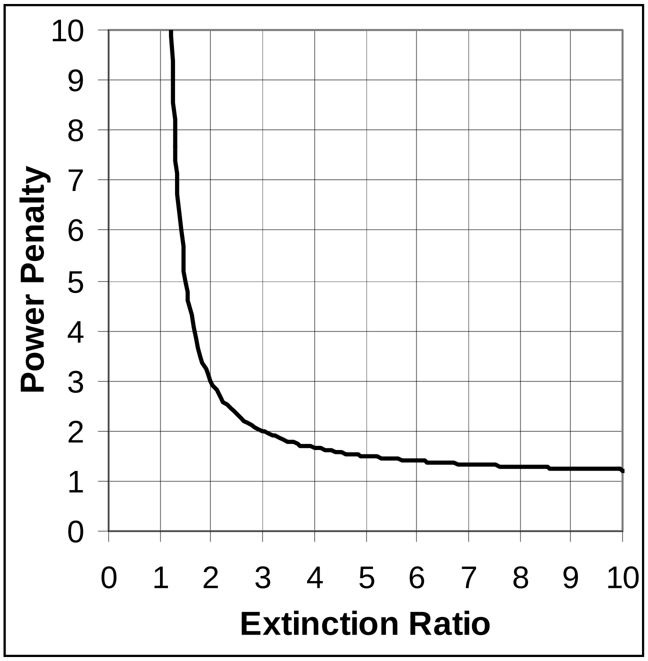

Figure 3. Power penalty vs. extinction ratio (linear ratios)

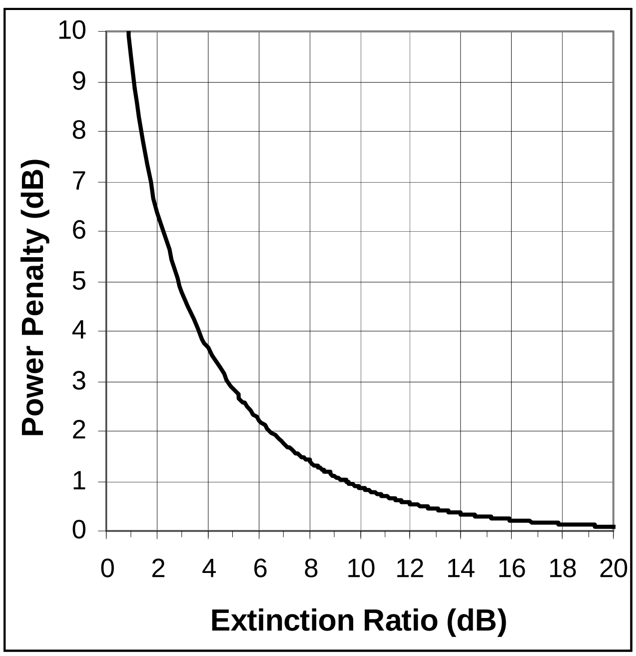

Figure 4. Power penalty vs. extinction ratio (dB ratios)

Table 1. Extinction Ratio and Power Penalty

Extinction Ratio		*Power Penalty*	
r_e	r_e (dB)	δ_e	δ_e (dB)
2.0	3.0	3.0	4.8
3.0	4.8	2.0	3.0
4.0	6.0	1.7	2.2
4.4	6.4	1.6	2.0
5.0	7.0	1.5	1.8
6.0	7.8	1.4	1.5
7.0	8.5	1.3	1.3
8.0	9.0	1.3	1.1
8.7	9.4	1.3	1.0
9.0	9.5	1.3	0.97
10.0	10.0	1.2	0.87
20.0	13.0	1.1	0.43
∞	∞	1.0	0.0

5 Conclusion

Seemingly small changes in extinction ratio can make a relatively large difference in the power required to maintain a constant BER. This effect is especially acute for extinction ratios less than seven, where a change of one in extinction ratio translates to an approximate 10% change in required average power. This additional required power is aptly termed the "power penalty," as nothing is gained by this increase in power other than the unnecessary privilege of operating at a reduced extinction ratio. Also, the DC offset associated with a low extinction ratio can cause overload problems in the receiver.

1. J. M. Senior, *Optical Fiber Communications: Principles and Practice*, Hertfordshire (UK): Prentice Hall International (UK) Ltd, pp. 625-627, 1992.

2. N. S. Bergano, F. W. Kerfoot, and C. R. Davidson, "Margin Measurements in Optical Amplifier Systems," in *IEEE Photonics Technology Letters,* vol. 5, no. 3, pp. 304-306, Mar. 1993.

3. G. Agrawal, *Fiber-Optic Communication Systems,* New York, N.Y.: John Wiley & Sons, pp. 172, 1997.