ANALOG DEVICES

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APPLICATION NOTE

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Using the Analog Devices Active Filter Design Tool

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INTRODUCTION

The Analog Devices Active Filter Design Tool assists the engineer in designing all-pole active filters.

The filter design process consists of two steps. In Step 1, the response of the filter is determined, meaning the attenuation and/or phase response of the filter is defined. In Step 2, the topology of the filter—how it is built—is defined. This application note is intended to help in Step 1. Several different standard responses are discussed, and their attenuation, group delay, step response, and impulse response are presented. The filter tool is then employed to design the filter. An example is provided.

STANDARD RESPONSES

Many transfer functions may be used to satisfy the attenuation and/or phase requirements of a particular filter. The one that is selected will depend on the particular system. The importance of frequency domain response versus time domain response must be determined. Also, both of these might be traded off against filter complexity, and therefore cost.

BUTTERWORTH FILTER

The Butterworth filter is the best compromise between attenuation and phase response. It has no ripple in the pass band or the stop band; because of this, it is sometimes called a maximally flat filter. The Butterworth filter achieves its flatness at the expense of a relatively wide transition region from pass band to stop band, with average transient characteristics.

The values of the elements of the Butterworth filter are more practical and less critical than many other filter types. The frequency response, group delay, impulse response, and step response are shown in Figure 1. The pole locations and corresponding ω_o and α terms are tabulated in Table II.

CHEBYSHEV FILTER

The Chebyshev (or Chevyshev, Tschebychev, Tschebyscheff, or Tchevysheff, depending on the translation from Russian) filter has a smaller transition region than the same-order Butterworth filter, at the expense of ripples in its pass band. This filter gets its name from the Chebyshev criterion, which minimizes the height of the maximum ripple.

Chebyshev filters have 0 dB relative attenuation at dc. Odd-order filters have an attenuation band that extends from 0 dB to the ripple value. Even-order filters have a gain equal to the pass-band ripple. The number of cycles of ripple in the pass band is equal to the order of the filter.

The Chebyshev filters are typically normalized so that the edge of the ripple band is at $\omega_o = 1$.

The 3 dB bandwidth is given by

$$A_{3dB} = \frac{1}{n}\cosh^{-1}\left(\frac{1}{\varepsilon}\right) \tag{1}$$

This is tabulated in Table I.

Figures 2 through 6 show the frequency response, group delay, impulse response, and step response for the various Chebyshev filters. The pole locations and corresponding ω_o and α terms are tabulated in Tables III through VII.

Table I. Chebyshev Cutoff Frequency to –3 dB Frequency

ORDER	0.01dB	0.1dB	0.25dB	0.5dB	1dB
2	3.30362	1.93432	1.59814	1.38974	1.21763
3	1.87718	1.38899	1.25289	1.16749	1.09487
4	1.46690	1.21310	1.13977	1.09310	1.05300
5	1.29122	1.13472	1.08872	1.05926	1.03381
6	1.19941	1.09293	1.06134	1.04103	1.02344
7	1.14527	1.06800	1.04495	1.03009	1.01721
8	1.11061	1.05193	1.03435	1.02301	1.01316
9	1.08706	1.04095	1.02711	1.01817	1.01040
10	1.07033	1.03313	1.02194	1.01471	1.00842

BESSEL FILTER

Butterworth filters have fairly good amplitude and transient behavior. The Chebyshev filters improve on the amplitude response at the expense of transient behavior. The Bessel filter is optimized to obtain better transient response due to a linear phase (i.e., constant delay) in the pass band. This means that there will be relatively poor frequency response (less amplitude discrimination).

The frequency response, group delay, impulse response, and step response for the Bessel filter are shown in Figure 7. The pole locations and corresponding ω_o and α terms are tabulated in Table VIII.

LINEAR PHASE WITH EQUIRIPPLE ERROR

The linear phase filter offers linear phase response in the pass band, over a wider range than the Bessel, and superior attenuation far from cutoff. This is accomplished by letting the phase response have ripples, similar to the amplitude ripples of the Chebyshev. As the ripple is increased, the region of constant delay extends further into the stop band. This will also cause the group delay to develop ripples, since it is the derivative of the phase response. The step response will show slightly more overshoot than the Bessel and the impulse response will show a bit more ringing.

The frequency response, group delay, impulse response, and step response for equiripple filters with error of 0.05° and 0.5° are shown in Figures 8 and 9, respectively. The pole locations and corresponding ω_o and α terms are tabulated in Tables IX and X.

GUASSIAN-TO-6 dB AND GUASSIAN-TO-12 dB FILTER

Gaussian-to-6 dB and Gaussian-to-12 dB filters are a compromise between a Chebyshev filter and a Gaussian filter, which is similar to a Bessel filter. A transitional filter has nearly linear phase shift and smooth, monotonic roll-off in the pass band. Above the pass band and especially at higher values of n, there is a break point beyond which the attenuation increases dramatically compared to that of the Bessel.

The Gaussian-to-6 dB filter has better transient response in the pass band than does the Butterworth filter. Beyond the breakpoint, which occurs at $\omega_o = 1.5$, the roll-off is similar to that of the Butterworth filter.

The Gaussian-to-12 dB filter's transient response in the pass band is much better than that of the Butterworth filter. Beyond the 12 dB breakpoint, which occurs at $\omega_o = 2$, the attenuation is less than that of the Butterworth filter.

The frequency response, group delay, impulse response, and step response for Gaussian-to-6 dB and Gaussian-to-12 dB filters are shown in Figures 10 and 11, respectively. The pole locations and corresponding ω_o and α terms are tabulated in Tables XI and XII.

USING THE PROTOTYPE RESPONSE CURVES

The response curves and design tables for several of the low-pass prototypes of the all-pole responses discussed previously are now cataloged. All of the curves are normalized to a –3 dB cutoff frequency of 1 Hz. This allows direct comparison of the various responses. In all cases, the amplitude response for the 2- through 10-pole cases for the frequency range of 0.1 Hz to 10 Hz will be shown. Then, a detail of the 0.1 Hz to 2 Hz pass band will be shown. The group delay from 0.1 Hz to 10 Hz, the impulse response, and the step response from 0 seconds to 5 seconds will also be shown.

Curves must be denormalized if they are to be used to determine the response of real life filters. In the case of the amplitude responses, this is accomplished by simply multiplying the frequency axis by the desired cutoff frequency, F_C. To denormalize the group delay curves, divide the delay axis by $2\pi F_C$ and multiply the frequency axis by F_C. Denormalize the step response by dividing the time axis by $2\pi F_C$. Denormalize the impulse response by dividing the time axis by $2\pi F_C$ and multiplying the amplitude axis by $2\pi F_C$.

For a high-pass filter, simply invert the frequency axis for the amplitude response. In transforming a low-pass filter into a high-pass filter, the transient behavior is not preserved. Zverev provides a computational method for calculating these responses.

In transforming a low-pass into a narrow-band band-pass, the 0 Hz axis is moved to the center frequency, F_0. It stands to reason that the response of the band-pass case around the center frequency would then match the low-pass response around 0 Hz. The frequency response curve of a low-pass filter actually mirrors itself around 0 Hz, although we generally do not concern ourselves with negative frequency.

To denormalize the group delay curve for a band-pass filter, divide the delay axis by πBW, where BW is the 3 dB bandwidth in Hz. Then, multiply the frequency axis by BW/2. In general, the delay of the band-pass filter at F_0 will be twice the delay of the low-pass prototype with the same bandwidth at 0 Hz. This is due to the fact that the low-pass to band-pass transformation results in a filter with order 2n, even though it is typically referred to as having the same order as the low-pass filter we derive it from. This approximation holds for narrow-band filters. As the bandwidth of the filter is increased, some distortion of the curve occurs. The delay becomes less symmetrical, peaking below F_0.

The envelope of the response of a band-pass filter resembles the step response of the low-pass prototype. More exactly, it is almost identical to the step response of a low-pass filter with half the bandwidth. To determine the envelope response of the band-pass filter, divide the time axis of the low-pass prototype's step response by πBW, where BW is the 3 dB bandwidth. The previous discussions of overshoot, ringing, and so on can now be applied to the carrier envelope.

The envelope of the response of a narrow-band band-pass filter to a short burst (where the burst width is much less than the rise time of the band-pass filter's denormalized step response) of carrier can be determined by denormalizing the impulse response of the low-pass prototype. To do this, multiply the amplitude axis and divide the time axis by πBW, where BW is the 3 dB bandwidth. It is assumed that the carrier frequency is high enough so that many cycles occur during the burst interval.

While the group delay, step, and impulse curves cannot be used directly to predict the distortion to the waveform caused by the filter, they are a useful figure of merit when used to compare filters.

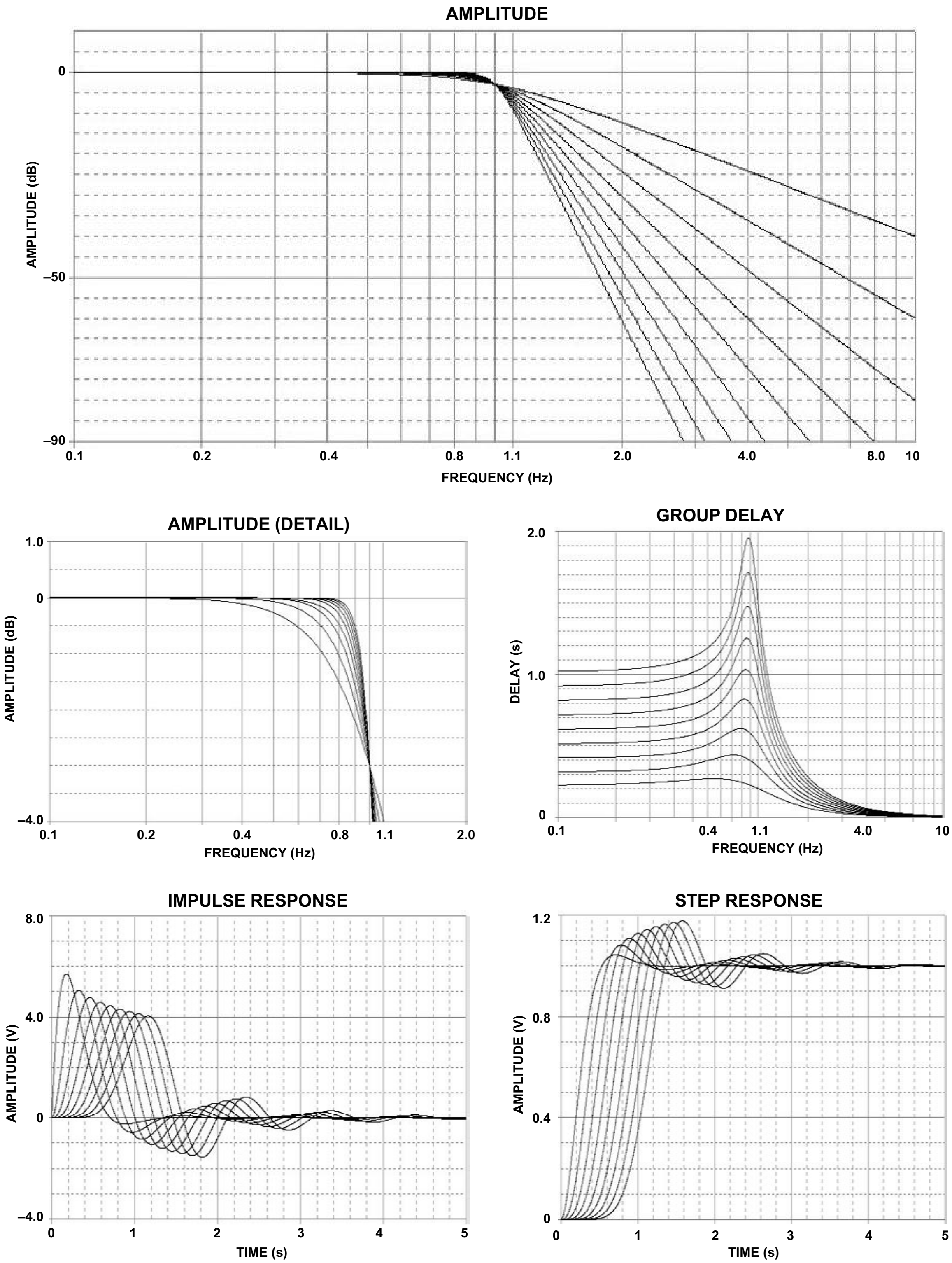


Figure 1. Butterworth Response

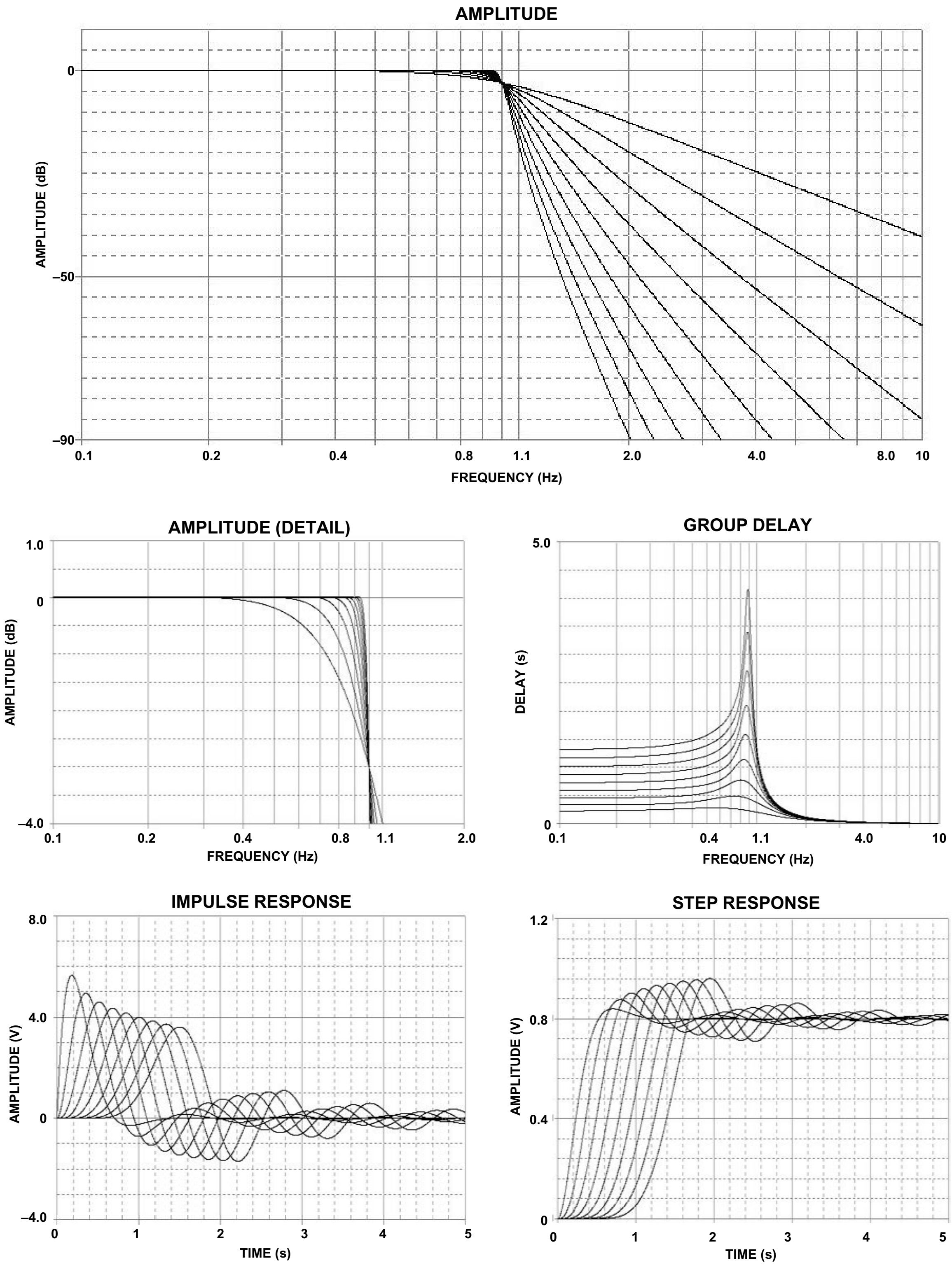


Figure 2. 0.01 dB Chebyshev Response

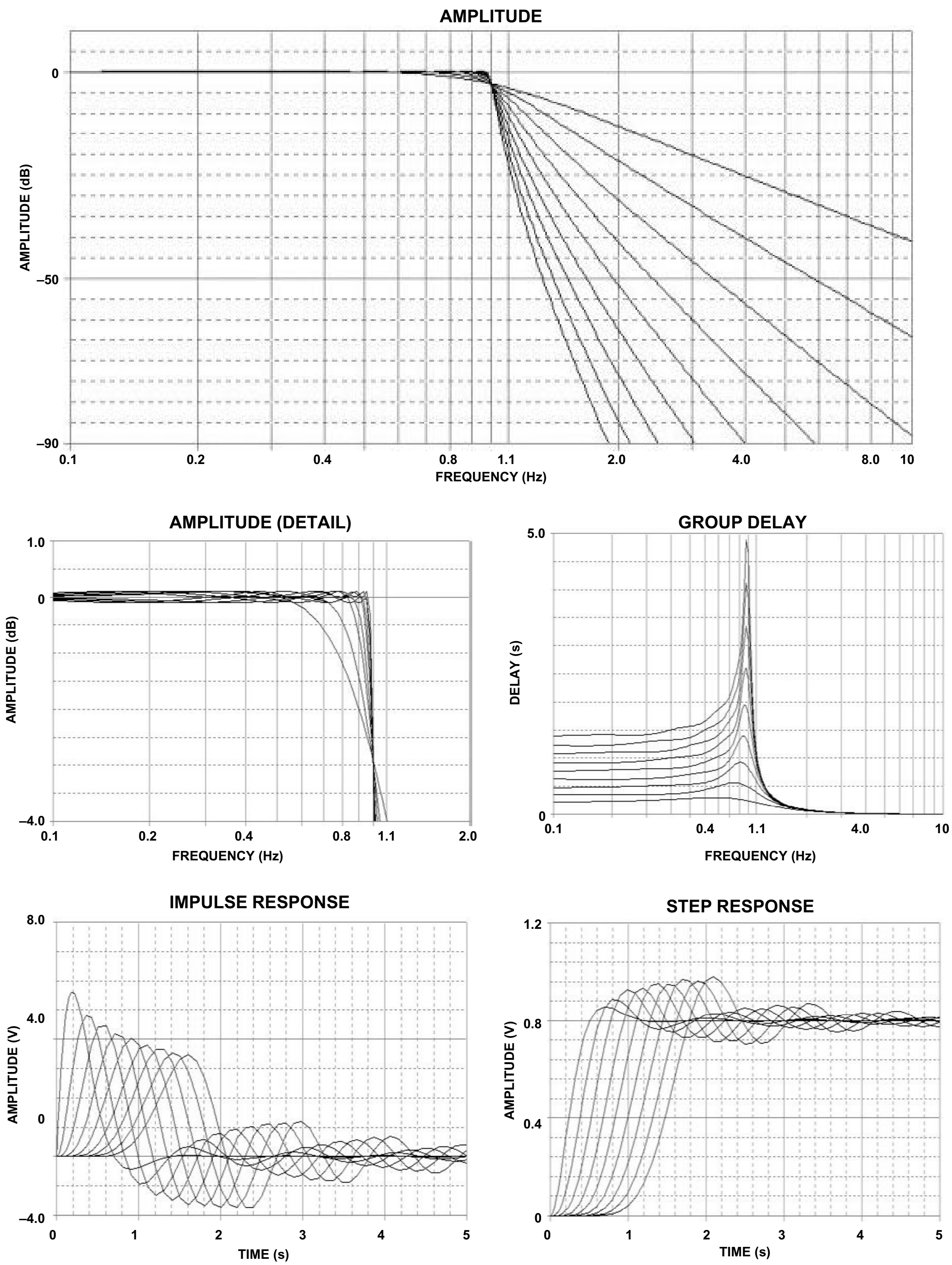


Figure 3. 0.1 dB Chebyshev Response

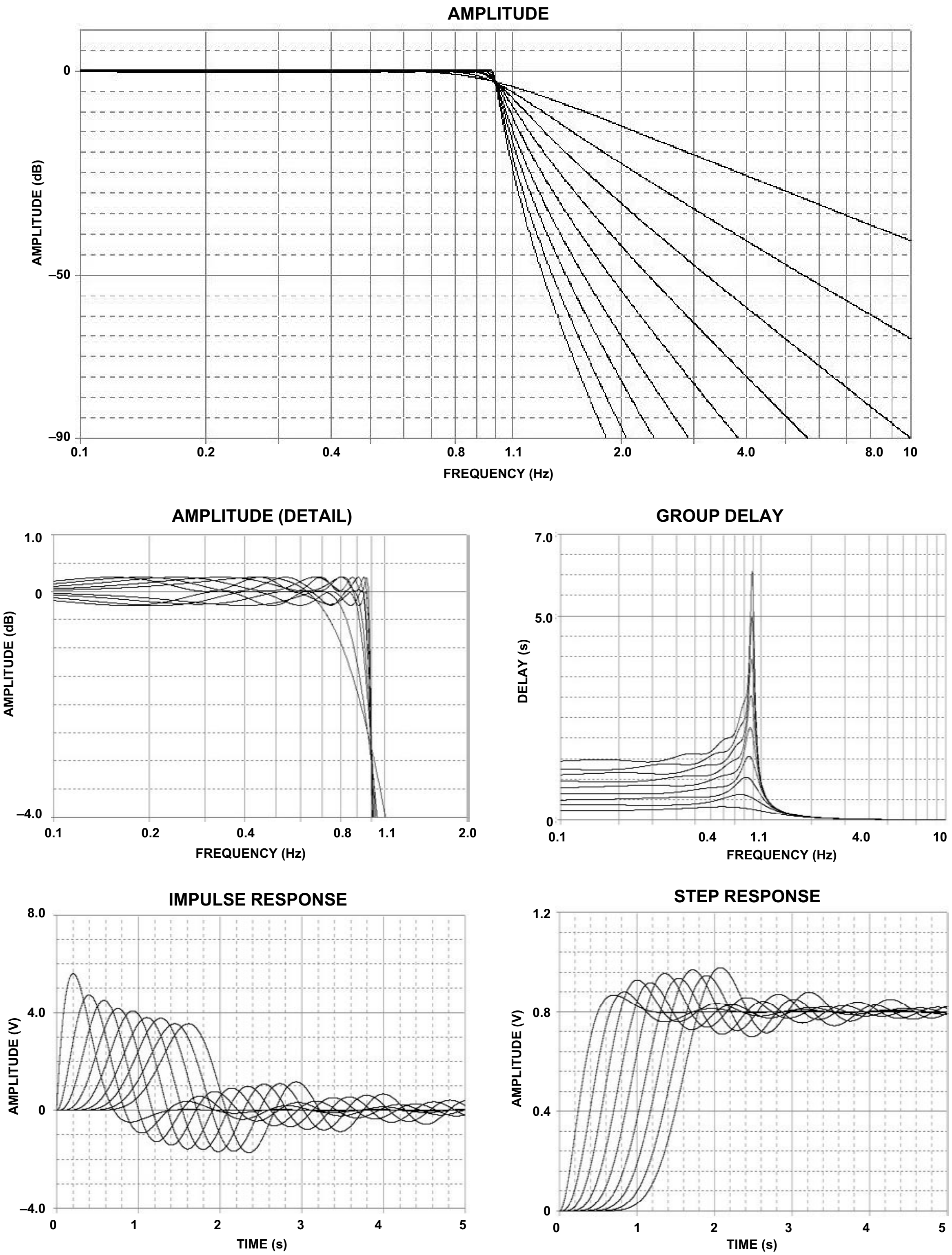


Figure 4. 0.25 dB Chebyshev Response

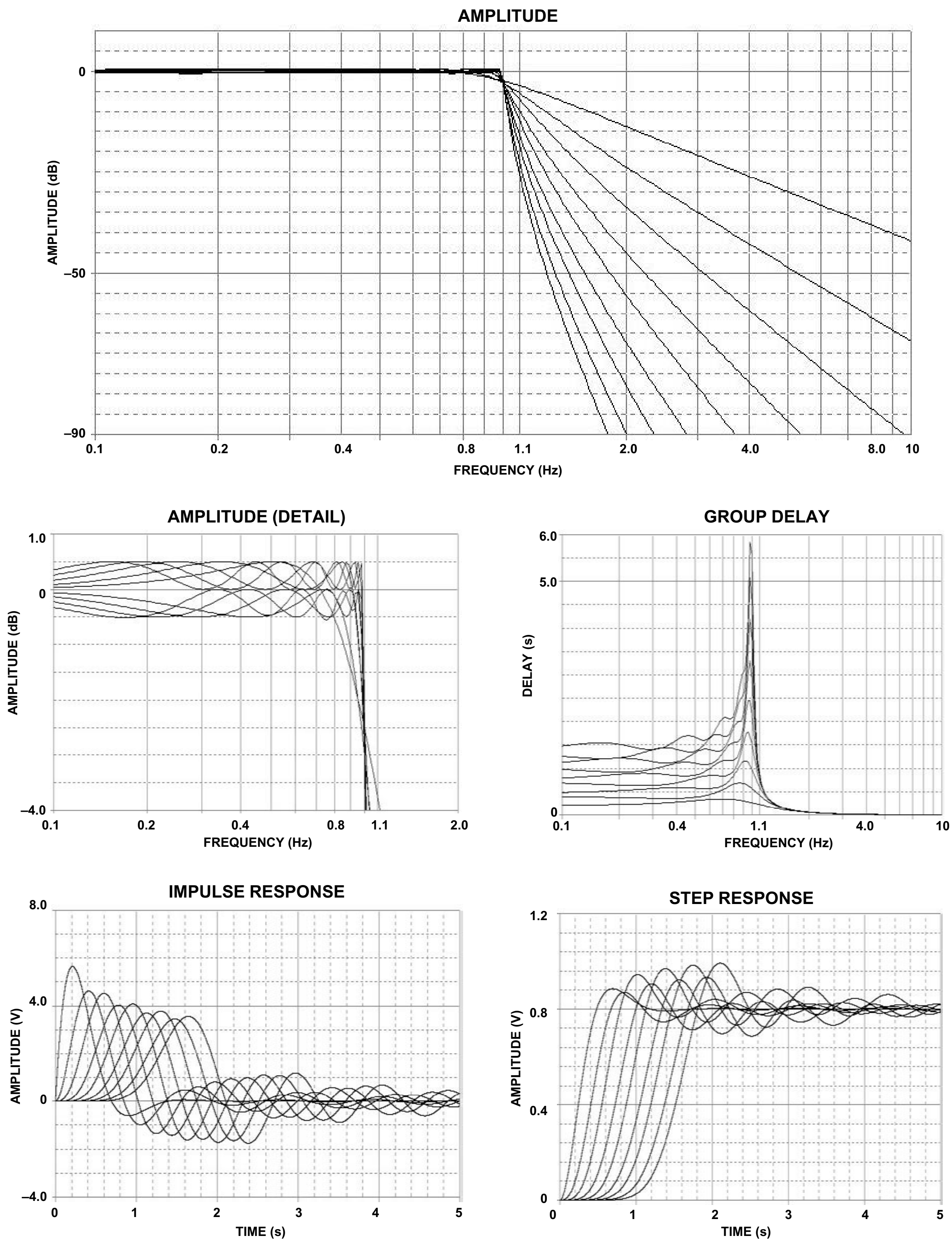


Figure 5. 0.5 dB Chebyshev Response

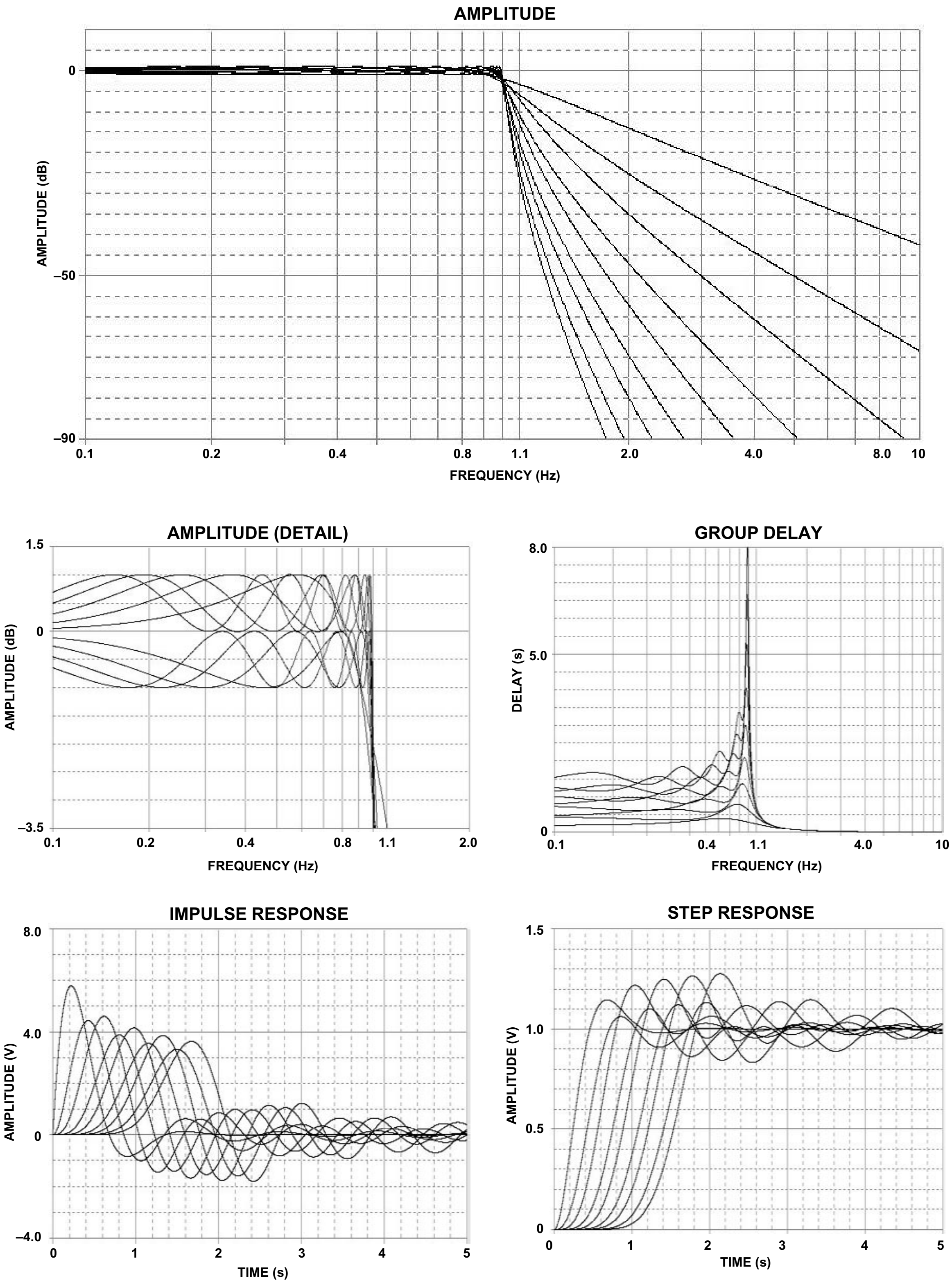


Figure 6. 1 dB Chebyshev Response

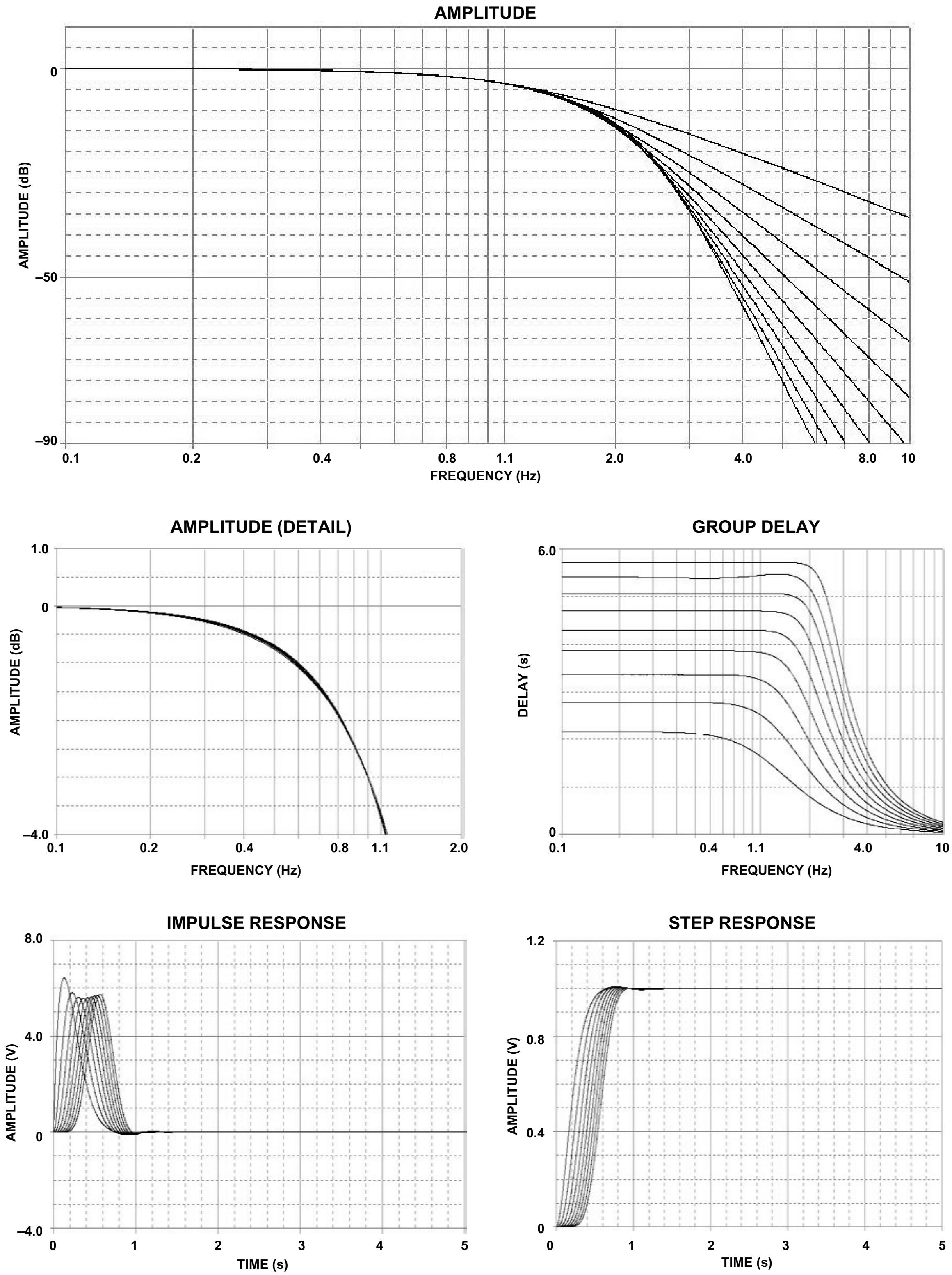


Figure 7. Bessel Response

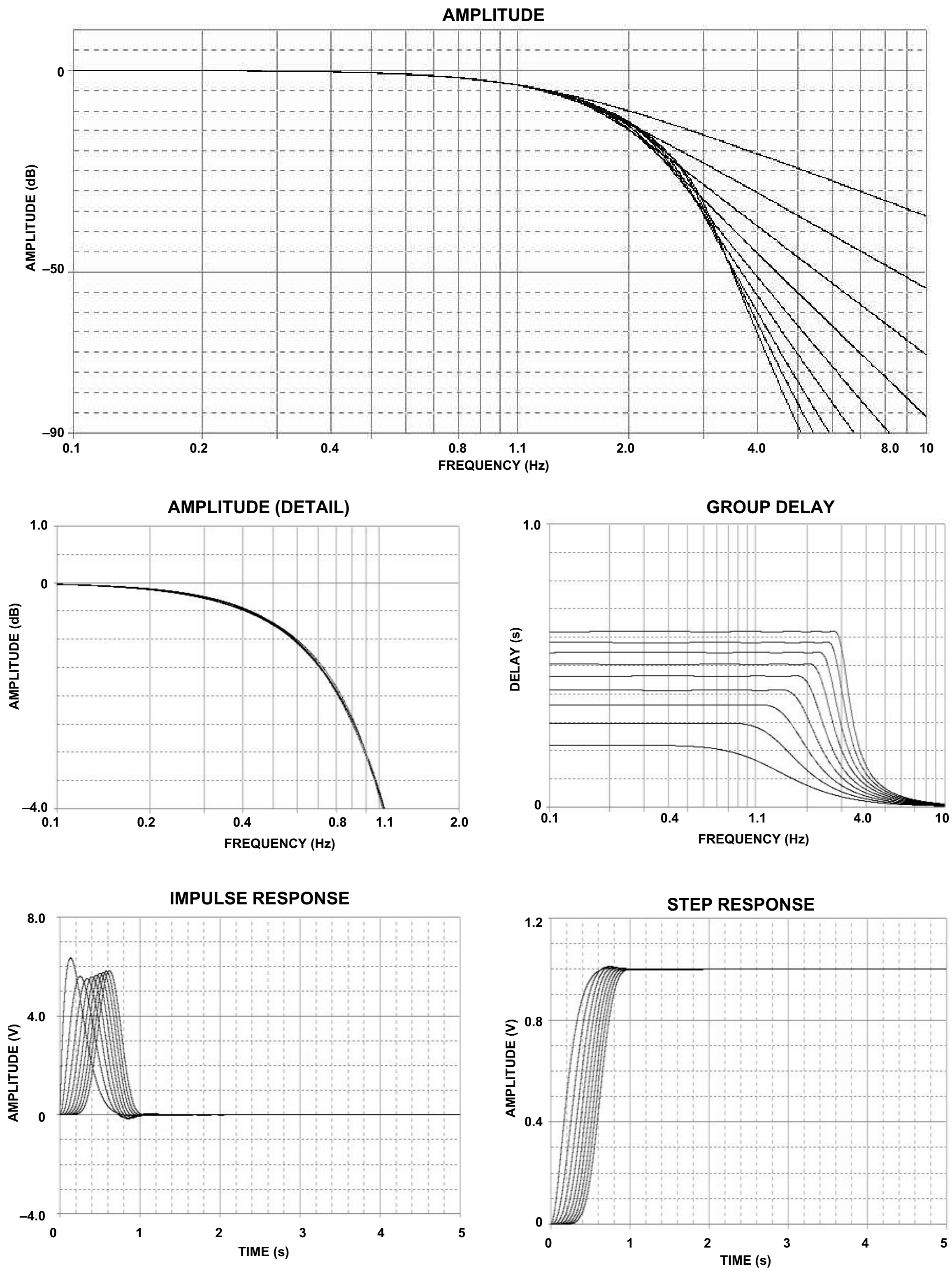


Figure 8. Linear Phase with Equiripple Error of 0.05° Response

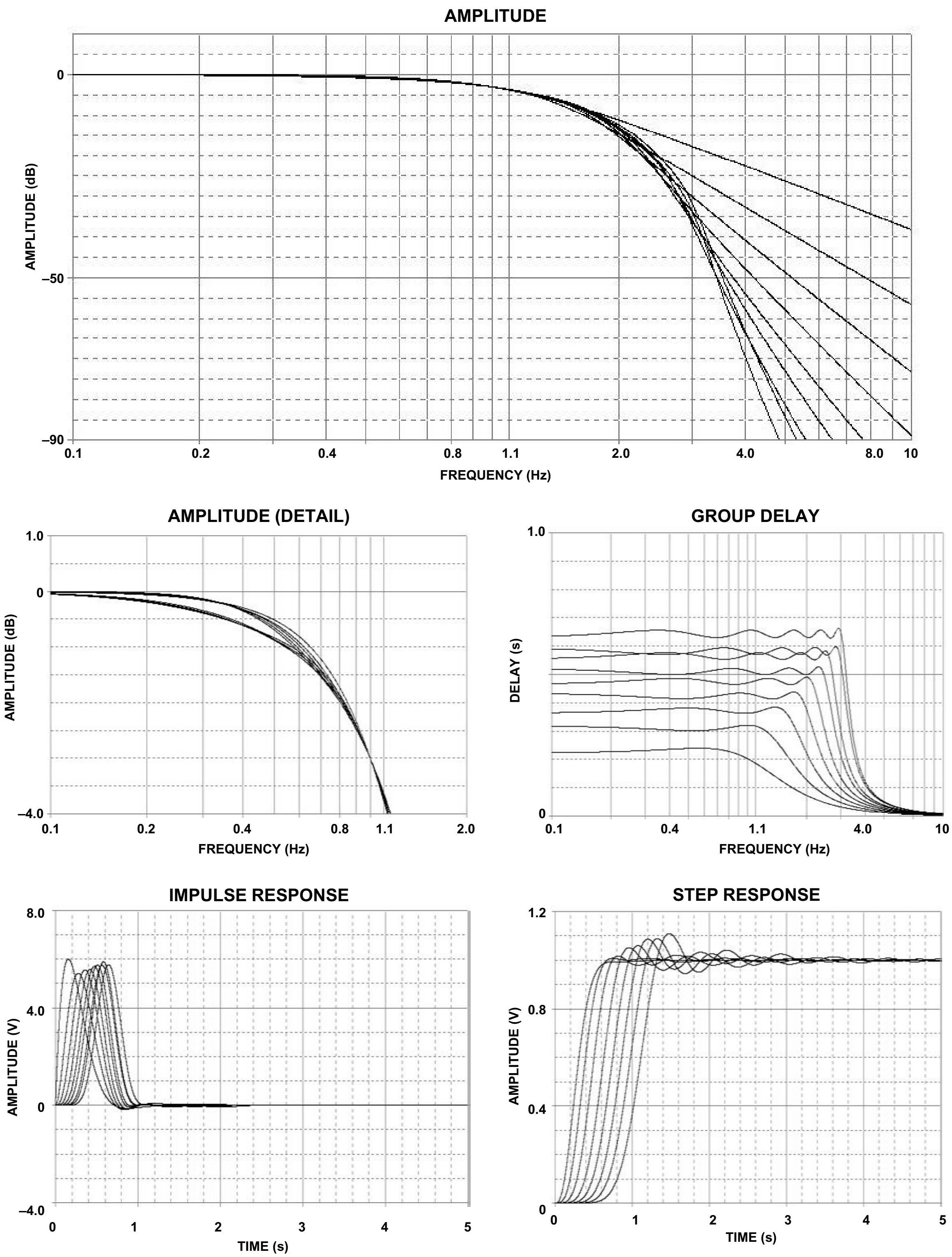


Figure 9. Linear Phase with Equiripple Error of 0.5° Response

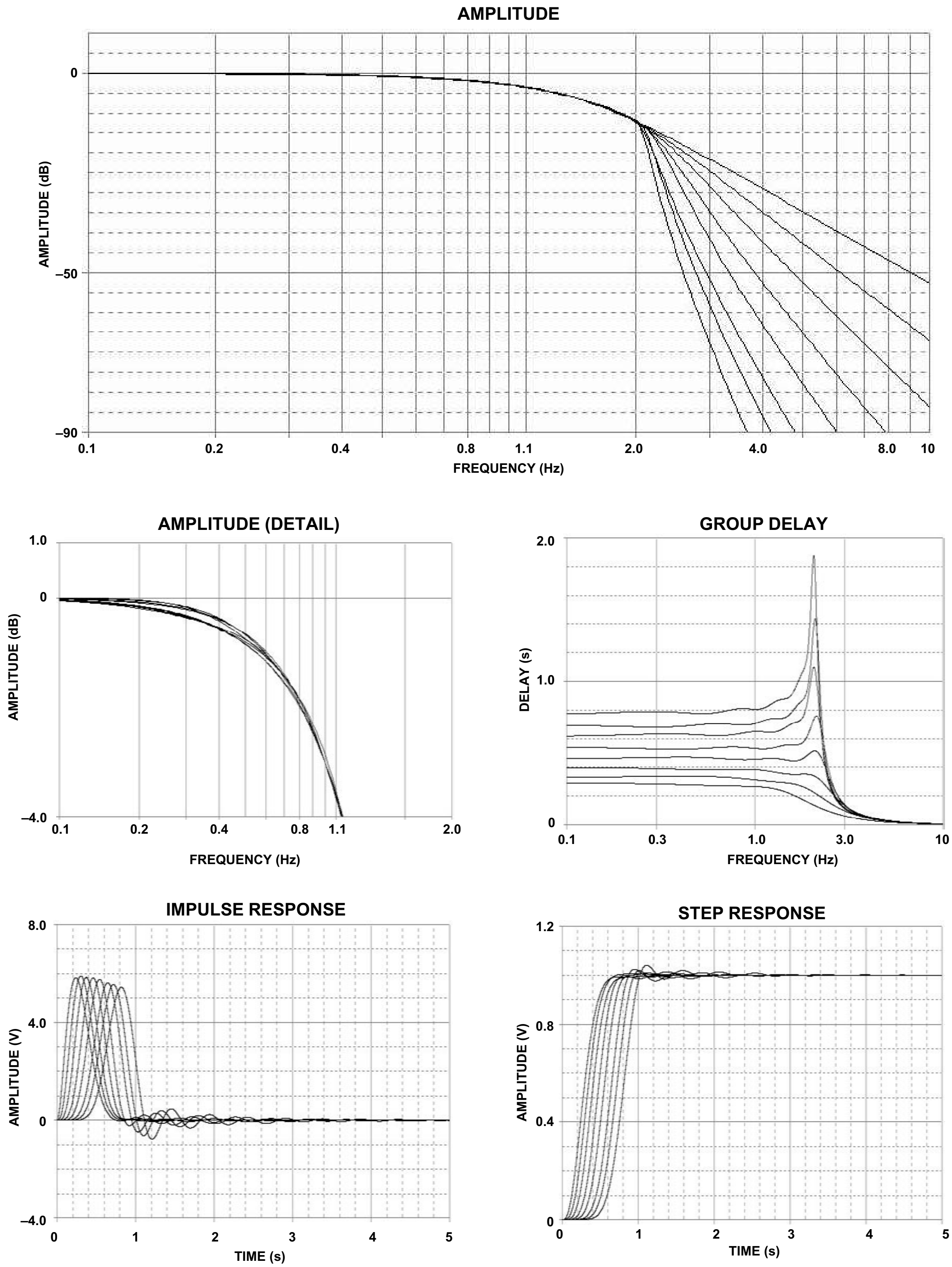


Figure 10. Gaussian-to-12 dB Response

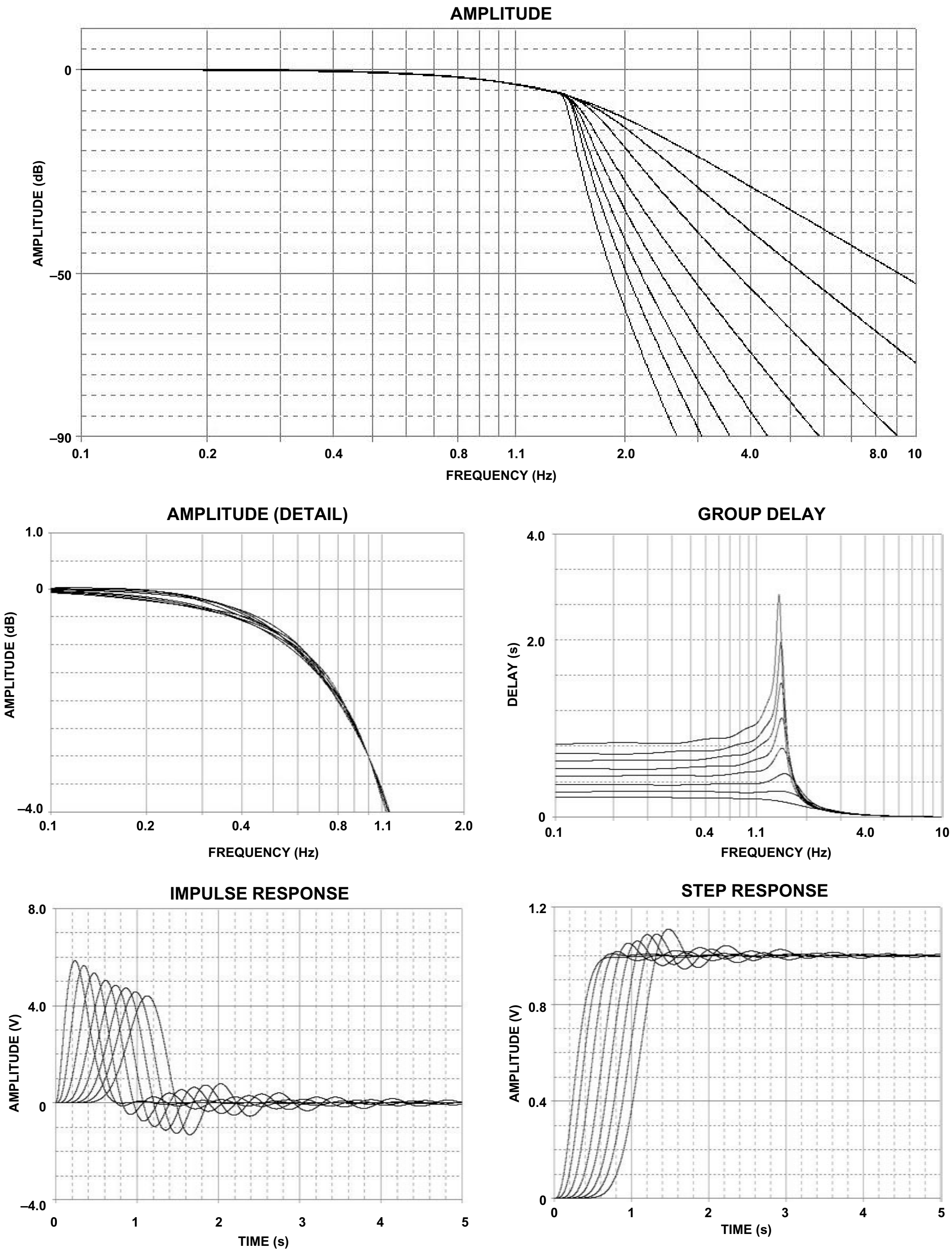


Figure 11. Gaussian-to-6 dB Response

Table II. Butterworth Design

ORDER	SECTION	REAL PART	IMAGINARY PART	F_O	α	Q	−3 dB FREQUENCY	PEAKING FREQUENCY	PEAKING LEVEL
2	1	0.7071	0.7071	1.0000	1.4142	0.7071	1.0000		
3	1	0.5000	0.8660	1.0000	1.0000	1.0000		0.7071	1.2493
	2	1.0000		1.0000			1.0000		
4	1	0.9239	0.3827	1.0000	1.8478	0.5412	0.7195		
	2	0.3827	0.9239	1.0000	0.7654	1.3065		0.8409	3.0102
5	1	0.8090	0.5878	1.0000	1.6180	0.6180	0.8588		
	2	0.3090	0.9511	1.0000	0.6180	1.6182		0.8995	4.6163
	3	1.0000		1.0000			1.0000		
6	1	0.9659	0.2588	1.0000	1.9319	0.5176	0.6758		
	2	0.7071	0.7071	1.0000	1.4142	0.7071	1.0000		
	3	0.2588	0.9659	1.0000	0.5176	1.9319		0.9306	6.0210
7	1	0.9010	0.4339	1.0000	1.8019	0.5550	0.7449		
	2	0.6235	0.7818	1.0000	1.2470	0.8019		0.4717	0.2204
	3	0.2225	0.9749	1.0000	0.4450	2.2471		0.9492	7.2530
	4	1.0000		1.0000			1.0000		
8	1	0.9808	0.1951	1.0000	1.9616	0.5098	0.6615		
	2	0.8315	0.5556	1.0000	1.6629	0.6013	0.8295		
	3	0.5556	0.8315	1.0000	1.1112	0.9000		0.6186	0.6876
	4	0.1951	0.9808	1.0000	0.3902	2.5628		0.9612	8.3429
9	1	0.9397	0.3420	1.0000	1.8794	0.5321	0.7026		
	2	0.7660	0.6428	1.0000	1.5320	0.6527	0.9172		
	3	0.5000	0.8660	1.0000	1.0000	1.0000		0.7071	1.2493
	4	0.1737	0.9848	1.0000	0.3474	2.8785		0.9694	9.3165
	5	1.0000		1.0000			1.0000		
10	1	0.9877	0.1564	1.0000	1.9754	0.5062	0.6549		
	2	0.8910	0.4540	1.0000	1.7820	0.5612	0.7564		
	3	0.7071	0.7071	1.0000	1.4142	0.7071	1.0000		
	4	0.4540	0.8910	1.0000	0.9080	1.1013		0.7667	1.8407
	5	0.1564	0.9877	1.0000	0.3128	3.1970		0.9752	10.2023

Table III. 0.01 dB Chebyshev Design

ORDER	SECTION	REAL PART	IMAGINARY PART	F_O	α	Q	−3 dB FREQUENCY	PEAKING FREQUENCY	PEAKING LEVEL
2	1	0.6743	0.7075	0.9774	1.3798	0.7247		0.2142	0.0100
3	1	0.4233	0.8663	0.9642	0.8780	1.1389		0.7558	2.0595
	2	0.8467		0.8467			0.8467		
4	1	0.6762	0.3828	0.7770	1.7405	0.5746	0.6069		
	2	0.2801	0.9241	0.9656	0.5801	1.7237		0.8806	5.1110
5	1	0.5120	0.5879	0.7796	1.3135	0.7613		0.2889	0.0827
	2	0.1956	0.9512	0.9711	0.4028	2.4824		0.9309	8.0772
	3	0.6328		0.6328			0.6328		
6	1	0.5335	0.2588	0.5930	1.7995	0.5557	0.4425		
	2	0.3906	0.7072	0.8079	0.9670	1.0342		0.5895	1.4482
	3	0.1430	0.9660	0.9765	0.2929	3.4144		0.9554	10.7605
7	1	0.4393	0.4339	0.6175	1.4229	0.7028	0.6136		
	2	0.3040	0.7819	0.8389	0.7247	1.3798		0.7204	3.4077
	3	0.1085	0.9750	0.9810	0.2212	4.5208		0.9689	13.1578
	4	0.4876		0.4876			0.4876		
8	1	0.4268	0.1951	0.4693	1.8190	0.5498	0.3451		
	2	0.3168	0.5556	0.6396	0.9907	1.0094		0.4564	1.3041
	3	0.2418	0.8315	0.8659	0.5585	1.7906		0.7956	5.4126
	4	0.0849	0.9808	0.9845	0.1725	5.7978		0.9771	15.2977
9	1	0.3686	0.3420	0.5028	1.4661	0.6821	0.4844		
	2	0.3005	0.6428	0.7096	0.8470	1.1807		0.5682	2.3008
	3	0.1961	0.8661	0.8880	0.4417	2.2642		0.8436	7.3155
	4	0.0681	0.9848	0.9872	0.1380	7.2478		0.9824	17.2249
	5	0.3923		0.3923			0.3923		
10	1	0.3522	0.1564	0.3854	1.8279	0.5471	0.2814		
	2	0.3178	0.454	0.5542	1.1469	0.8719		0.3242	0.5412
	3	0.2522	0.7071	0.7507	0.6719	1.4884		0.6606	3.9742
	4	0.1619	0.891	0.9056	0.3576	2.7968		0.8762	9.0742
	5	0.0558	0.9877	0.9893	0.1128	8.8645		0.9861	18.9669

Table IV. 0.1 dB Chebyshev Design

ORDER	SECTION	REAL PART	IMAGINARY PART	F_0	α	Q	−3 dB FREQUENCY	PEAKING FREQUENCY	PEAKING LEVEL
2	1	0.6104	0.7106	0.9368	1.3032	0.7673		0.3638	0.0999
3	1	0.3490	0.8684	0.9359	0.7458	1.3408		0.7952	3.1978
	2	0.6970		0.6970			0.6970		
4	1	0.2177	0.9254	0.9507	0.4580	2.1834		0.8994	7.0167
	2	0.5257	0.3833	0.6506	1.6160	0.6188	0.5596		
5	1	0.3842	0.5884	0.7027	1.0935	0.9145		0.4457	0.7662
	2	0.1468	0.9521	0.9634	0.3048	3.2812		0.9407	10.4226
	3	0.4749		0.4749			0.4749		
6	1	0.3916	0.2590	0.4695	1.6682	0.5995	0.3879		
	2	0.2867	0.7077	0.7636	0.7509	1.3316		0.6470	3.1478
	3	0.1049	0.9667	0.9724	0.2158	4.6348		0.9610	13.3714
7	1	0.3178	0.4341	0.5380	1.1814	0.8464		0.2957	0.4157
	2	0.2200	0.7823	0.8126	0.5414	1.8469		0.7507	5.6595
	3	0.0785	0.9755	0.9787	0.1604	6.2335		0.9723	15.9226
	4	0.3528		0.3528			0.3528		
8	1	0.3058	0.1952	0.3628	1.6858	0.5932	0.2956		
	2	0.2529	0.5558	0.6106	0.8283	1.2073		0.4949	2.4532
	3	0.1732	0.8319	0.8497	0.4077	2.4531		0.8137	7.9784
	4	0.0608	0.9812	0.9831	0.1237	8.0819		0.9793	18.1669
9	1	0.2622	0.3421	0.4310	1.2166	0.8219		0.2197	0.3037
	2	0.2137	0.6430	0.6776	0.6308	1.5854		0.6064	4.4576
	3	0.1395	0.8663	0.8775	0.3180	3.1450		0.8550	10.0636
	4	0.0485	0.9852	0.9864	0.0982	10.1795		0.9840	20.1650
	5	0.2790		0.2790			0.2790		
10	1	0.2493	0.1564	0.2943	1.6942	0.5902	0.2382		
	2	0.2249	0.4541	0.5067	0.8876	1.1266		0.3945	1.9880
	3	0.1785	0.7073	0.7295	0.4894	2.0434		0.6844	6.4750
	4	0.1146	0.8913	0.8986	0.2551	3.9208		0.8839	11.9386
	5	0.0395	0.9880	0.9888	0.0799	12.5163		0.9872	21.9565

Table V. 0.25 dB Chebyshev Design

ORDER	SECTION	REAL PART	IMAGINARY PART	F_0	α	Q	−3 dB FREQUENCY	PEAKING FREQUENCY	PEAKING LEVEL
2	1	0.5621	0.7154	0.9098	1.2356	0.8093		0.4425	0.2502
3	1	0.3062	0.8712	0.9234	0.6632	1.5079		0.8156	4.0734
	2	0.6124		0.6124			0.6124		
4	1	0.4501	0.3840	0.5916	1.5215	0.6572	0.5470		
	2	0.1865	0.9272	0.9458	0.3944	2.5356		0.9082	8.2538
5	1	0.3247	0.5892	0.6727	0.9653	1.0359		0.4917	1.4585
	2	0.1240	0.9533	0.9613	0.2580	3.8763		0.9452	11.8413
	3	0.4013		0.4013			0.4013		
6	1	0.3284	0.2593	0.4184	1.5697	0.6371	0.3730		
	2	0.2404	0.7083	0.7480	0.6428	1.5557		0.6663	4.3121
	3	0.0880	0.9675	0.9715	0.1811	5.5205		0.9635	14.8753
7	1	0.2652	0.4344	0.5090	1.0421	0.9596		0.3441	1.0173
	2	0.1835	0.7828	0.8040	0.4565	2.1908		0.7610	7.0443
	3	0.0655	0.9761	0.9783	0.1339	7.4679		0.9739	17.4835
	4	0.2944		0.2944			0.2944		
8	1	0.2543	0.1953	0.3206	1.5862	0.6304	0.2822		
	2	0.2156	0.5561	0.5964	0.7230	1.3832		0.5126	3.4258
	3	0.1441	0.8323	0.8447	0.3412	2.9309		0.8197	9.4683
	4	0.0506	0.9817	0.9830	0.1029	9.7173		0.9804	19.7624
9	1	0.2176	0.3423	0.4056	1.0730	0.9320		0.2642	0.8624
	2	0.1774	0.6433	0.6673	0.5317	1.8808		0.6184	5.8052
	3	0.1158	0.8667	0.8744	0.2649	3.7755		0.8589	11.6163
	4	0.0402	0.9856	0.9864	0.0815	12.2659		0.9848	21.7812
	5	0.2315		0.2315			0.2315		
10	1	0.2065	0.1565	0.2591	1.5940	0.6274	0.2267		
	2	0.1863	0.4543	0.4910	0.7588	1.3178		0.4143	3.0721
	3	0.1478	0.7075	0.7228	0.4090	2.4451		0.6919	7.9515
	4	0.0949	0.8915	0.8965	0.2117	4.7236		0.8864	13.5344
	5	0.0327	0.9883	0.9888	0.0661	15.1199		0.9878	23.5957

Table VI. 0.5 dB Chebyshev Design

ORDER	SECTION	REAL PART	IMAGINARY PART	F_0	α	Q	−3 dB FREQUENCY	PEAKING FREQUENCY	PEAKING LEVEL
2	1	0.5129	0.7225	1.2314	1.1577	0.8638		0.7072	0.5002
3	1	0.2683	0.8753	1.0688	0.5861	1.7061		0.9727	5.0301
	2	0.5366		0.6265			0.6265		
4	1	0.3872	0.3850	0.5969	1.4182	0.7051	0.5951		
	2	0.1605	0.9297	1.0313	0.3402	2.9391		1.0010	9.4918
5	1	0.2767	0.5902	0.6905	0.8490	1.1779		0.5522	2.2849
	2	0.1057	0.9550	1.0178	0.2200	4.5451		1.0054	13.2037
	3	0.3420		0.3623			0.3623		
6	1	0.2784	0.2596	0.3963	1.4627	0.6836	0.3827		
	2	0.2037	0.7091	0.7680	0.5522	1.8109		0.7071	5.5025
	3	0.0746	0.9687	1.0114	0.1536	6.5119		1.0055	16.2998
7	1	0.2241	0.4349	0.5040	0.9161	1.0916		0.3839	1.7838
	2	0.1550	0.7836	0.8228	0.3881	2.5767		0.7912	8.3880
	3	0.0553	0.9771	1.0081	0.1130	8.8487		1.0049	18.9515
	4	0.2487		0.2562			0.2562		
8	1	0.2144	0.1955	0.2968	1.4779	0.6767	0.2835		
	2	0.1817	0.5565	0.5989	0.6208	1.6109		0.5381	4.5815
	3	0.1214	0.8328	0.8610	0.2885	3.4662		0.8429	10.8885
	4	0.0426	0.9824	1.0060	0.0867	11.5305		1.0041	21.2452
9	1	0.1831	0.3425	0.3954	0.9429	1.0605		0.2947	1.6023
	2	0.1493	0.6436	0.6727	0.4520	2.2126		0.6374	7.1258
	3	0.0974	0.8671	0.8884	0.2233	4.4779		0.8773	13.0759
	4	0.0338	0.9861	1.0046	0.0686	14.5829		1.0034	23.2820
	5	0.1949		0.1984			0.1984		
10	1	0.1736	0.1566	0.2338	1.4851	0.6734	0.2221		
	2	0.1566	0.4545	0.4807	0.6515	1.5349		0.4267	4.2087
	3	0.1243	0.7078	0.7186	0.3459	2.8907		0.6968	9.3520
	4	0.0798	0.8919	0.8955	0.1782	5.6107		0.8883	15.0149
	5	0.0275	0.9887	0.9891	0.0556	17.9833		0.9883	25.1008

Table VII. 1 dB Chebyshev Design

ORDER	SECTION	REAL PART	IMAGINARY PART	F_0	α	Q	−3 dB FREQUENCY	PEAKING FREQUENCY	PEAKING LEVEL
2	1	0.4508	0.7351	0.8623	1.0456	0.9564		0.5806	0.9995
3	1	0.2257	0.8822	0.9106	0.4957	2.0173		0.8528	6.3708
	2	0.4513		0.4513			0.4513		
4	1	0.3199	0.3868	0.5019	1.2746	0.7845		0.2174	0.1557
	2	0.1325	0.9339	0.9433	0.2809	3.5594		0.9245	11.1142
5	1	0.2265	0.5918	0.6337	0.7149	1.3988		0.5467	3.5089
	2	0.0865	0.9575	0.9614	0.1800	5.5559		0.9536	14.9305
	3	0.2800		0.2800			0.2800		
6	1	0.2268	0.2601	0.3451	1.3144	0.7608		0.1273	0.0813
	2	0.1550	0.7106	0.7273	0.4262	2.3462		0.6935	7.6090
	3	0.0608	0.9707	0.9726	0.1249	8.0036		0.9688	18.0827
7	1	0.1819	0.4354	0.4719	0.7710	1.2971		0.3956	2.9579
	2	0.1259	0.7846	0.7946	0.3169	3.1558		0.7744	10.0927
	3	0.0449	0.9785	0.9795	0.0918	10.8982		0.9775	20.7563
	4	0.2019		0.2019			0.2019		
8	1	0.1737	0.1956	0.2616	1.3280	0.7530		0.0899	0.0611
	2	0.1473	0.5571	0.5762	0.5112	1.9560		0.5373	6.1210
	3	0.0984	0.8337	0.8395	0.2344	4.2657		0.8279	12.6599
	4	0.0346	0.9836	0.9842	0.0702	14.2391		0.9830	23.0750
9	1	0.1482	0.3427	0.3734	0.7938	1.2597		0.3090	2.7498
	2	0.1208	0.6442	0.6554	0.3686	2.7129		0.6328	8.8187
	3	0.0788	0.8679	0.8715	0.1809	5.5268		0.8643	14.8852
	4	0.0274	0.9869	0.9873	0.0555	18.0226		0.9865	25.1197
	5	0.1577		0.1577			0.1577		
10	1	0.1403	0.1567	0.2103	1.3341	0.7496		0.0698	0.0530
	2	0.1266	0.4548	0.4721	0.5363	1.8645		0.4368	5.7354
	3	0.1005	0.7084	0.7155	0.2809	3.5597		0.7012	11.1147
	4	0.0645	0.8926	0.8949	0.1441	6.9374		0.8903	16.8466
	5	0.0222	0.9895	0.9897	0.0449	22.2916		0.9893	26.9650

Table VIII. Bessel Design

ORDER	SECTION	REAL PART	IMAGINARY PART	F_0	α	Q	−3 dB FREQUENCY	PEAKING FREQUENCY	PEAKING LEVEL
2	1	1.1050	0.6368	1.2754	1.7328	0.5771	1.0020		
3	1	1.0509	1.0025	1.4524	1.4471	0.6910	1.4185		
	2	1.3270		1.3270			1.3270		
4	1	1.3596	0.4071	1.4192	1.9160	0.5219	0.9705		
	2	0.9877	1.2476	1.5912	1.2414	0.8055		0.7622	0.2349
5	1	1.3851	0.7201	1.5611	1.7745	0.5635	1.1876		
	2	0.9606	1.4756	1.7607	1.0911	0.9165		1.1201	0.7768
	3	1.5069		1.5069			1.5069		
6	1	1.5735	0.3213	1.6060	1.9596	0.5103	1.0638		
	2	1.3836	0.9727	1.6913	1.6361	0.6112	1.4323		
	3	0.9318	1.6640	1.9071	0.9772	1.0234		1.3786	1.3851
7	1	1.6130	0.5896	1.7174	1.8784	0.5324	1.2074		
	2	1.3797	1.1923	1.8235	1.5132	0.6608	1.6964		
	3	0.9104	1.8375	2.0507	0.8879	1.1262		1.5961	1.9860
	4	1.6853		1.6853			1.6853		
8	1	1.7627	0.2737	1.7838	1.9763	0.5060	1.1675		
	2	0.8955	2.0044	2.1953	0.8158	1.2258		1.7932	2.5585
	3	1.3780	1.3926	1.9591	1.4067	0.7109		0.2011	0.0005
	4	1.6419	0.8256	1.8378	1.7868	0.5597	1.3849		
9	1	1.8081	0.5126	1.8794	1.9242	0.5197	1.2774		
	2	1.6532	1.0319	1.9488	1.6966	0.5894	1.5747		
	3	1.3683	1.5685	2.0815	1.3148	0.7606		0.7668	0.0807
	4	0.8788	2.1509	2.3235	0.7564	1.3220		1.9632	3.0949
	5	1.8575		1.8575			1.8575		
10	1	1.9335	0.2451	1.9490	1.9841	0.5040	1.2685		
	2	1.8467	0.7335	1.9870	1.8587	0.5380	1.4177		
	3	1.6661	1.2246	2.0678	1.6115	0.6205	1.7848		
	4	1.3648	1.7395	2.2110	1.2346	0.8100		1.0785	0.2531
	5	0.8686	2.2994	2.4580	0.7067	1.4150		2.1291	3.5944

Table IX. Equiripple with 0.05° Error Design

ORDER	SECTION	REAL PART	IMAGINARY PART	F_0	α	Q	−3 dB FREQUENCY	PEAKING FREQUENCY	PEAKING LEVEL
2	1	1.0087	0.6680	1.2098	1.6675	0.5997	0.9999		
3	1	0.8541	1.0725	1.3710	1.2459	0.8026		0.6487	0.2232
	2	1.0459		1.0459			1.0459		
4	1	0.9648	0.4748	1.0753	1.7945	0.5573	0.8056		
	2	0.7448	1.4008	1.5865	0.9389	1.0650		1.1864	1.6286
5	1	0.8915	0.8733	1.2480	1.4287	0.6999	1.2351		
	2	0.6731	1.7085	1.8363	0.7331	1.3641		1.5703	3.3234
	3	0.9430		0.9430			0.9430		
6	1	0.8904	0.4111	0.9807	1.8158	0.5507	0.7229		
	2	0.8233	1.2179	1.4701	1.1201	0.8928		0.8975	0.6495
	3	0.6152	1.9810	2.0743	0.5932	1.6859		1.8831	4.9365
7	1	0.8425	0.7791	1.1475	1.4684	0.6810	1.1036		
	2	0.7708	1.5351	1.7177	0.8975	1.1143		1.3276	1.9162
	3	0.5727	2.2456	2.3175	0.4942	2.0233		2.1713	6.3948
	4	0.8615		0.8615			0.8615		
8	1	0.8195	0.3711	0.8996	1.8219	0.5489	0.6600		
	2	0.7930	1.1054	1.3604	1.1658	0.8578		0.7701	0.4705
	3	0.7213	1.8134	1.9516	0.7392	1.3528		1.6638	3.2627
	4	0.5341	2.4761	2.5330	0.4217	2.3713		2.4178	7.6973
9	1	0.7853	0.7125	1.0604	1.4812	0.6751	1.0102		
	2	0.7555	1.4127	1.6020	0.9432	1.0602		1.1937	1.6005
	3	0.6849	2.0854	2.1950	0.6241	1.6024		1.9697	4.5404
	4	0.5060	2.7133	2.7601	0.3667	2.7274		2.6657	8.8633
	5	0.7983		0.7983			0.7983		
10	1	0.7592	0.3413	0.8324	1.8241	0.5482	0.6096		
	2	0.7467	1.0195	1.2637	1.1818	0.8462		0.6941	0.4145
	3	0.7159	1.6836	1.8295	0.7826	1.2778		1.5238	2.8507
	4	0.6475	2.3198	2.4085	0.5377	1.8598		2.2276	5.7152
	5	0.4777	2.9128	2.9517	0.3237	3.0895		2.8734	9.9130

Table X. Equiripple with 0.5° Error Design

ORDER	SECTION	REAL PART	IMAGINARY PART	F_0	α	Q	–3 dB FREQUENCY	PEAKING FREQUENCY	PEAKING LEVEL
2	1	0.8590	0.6981	1.1069	1.5521	0.6443	1.0000		
3	1	0.6969	1.1318	1.3292	1.0486	0.9536		0.8918	0.9836
	2	0.8257		0.8257			0.8257		
4	1	0.7448	0.5133	0.9045	1.6468	0.6072	0.7597		
	2	0.6037	1.4983	1.6154	0.7475	1.3379		1.3713	3.1817
5	1	0.6775	0.9401	1.1588	1.1693	0.8552		0.6518	0.4579
	2	0.5412	1.8256	1.9041	0.5684	1.7592		1.7435	5.2720
	3	0.7056		0.7056			0.7056		
6	1	0.6519	0.4374	0.7850	1.6608	0.6021	0.6522		
	2	0.6167	1.2963	1.4355	0.8592	1.1639		1.1402	2.2042
	3	0.4893	2.0982	2.1545	0.4542	2.2016		2.0404	7.0848
7	1	0.6190	0.8338	1.0385	1.1922	0.8388		0.5586	0.3798
	2	0.5816	1.6455	1.7453	0.6665	1.5004		1.5393	4.0353
	3	0.4598	2.3994	2.4431	0.3764	2.6567		2.3549	8.6433
	4	0.6283		0.6283			0.6283		
8	1	0.5791	0.3857	0.6958	1.6646	0.6007	0.5764		
	2	0.5665	1.1505	1.2824	0.8835	1.1319		1.0014	2.0187
	3	0.5303	1.8914	1.9643	0.5399	1.8521		1.8155	5.6819
	4	0.4148	2.5780	2.6112	0.3177	3.1475		2.5444	10.0703
9	1	0.5688	0.7595	0.9489	1.1989	0.8341		0.5033	0.3581
	2	0.5545	1.5089	1.6076	0.6899	1.4496		1.4033	3.7748
	3	0.5179	2.2329	2.2922	0.4519	2.2130		2.1720	7.1270
	4	0.4080	2.9028	2.9313	0.2784	3.5923		2.8740	11.1925
	5	0.5728		0.5728			0.5728		
10	1	0.5249	0.3487	0.6302	1.6659	0.6003	0.5215		
	2	0.5193	1.0429	1.1650	0.8915	1.1217		0.9044	1.9598
	3	0.5051	1.7264	1.7988	0.5616	1.7806		1.6509	5.3681
	4	0.4711	2.3850	2.4311	0.3876	2.5802		2.3380	8.3994
	5	0.3708	2.9940	3.0169	0.2458	4.0681		2.9709	12.2539

Table XI. Gaussian-to-12 dB Design

ORDER	SECTION	REAL PART	IMAGINARY PART	F_0	α	Q	–3 dB FREQUENCY	PEAKING FREQUENCY	PEAKING LEVEL
3	1	0.9622	1.2214	1.5549	1.2377	0.8080		0.7523	0.2448
	2	0.9776	0.5029	1.0994	1.7785	0.5623	0.8338		
4	1	0.7940	0.5029	0.9399	1.6896	0.5919	0.7636		
	2	0.6304	1.5407	1.6647	0.7574	1.3203		1.4058	3.0859
5	1	0.6190	0.8254	1.0317	1.1999	0.8334		0.5460	0.3548
	2	0.3559	1.5688	1.6087	0.4425	2.2600		1.5279	7.3001
	3	0.6650		0.6650			0.6650		
6	1	0.5433	0.3431	0.6426	1.6910	0.5914	0.5215		
	2	0.4672	0.9991	1.1029	0.8472	1.1804		0.8831	2.2992
	3	0.2204	1.5067	1.5227	0.2895	3.4545		1.4905	10.8596
7	1	0.4580	0.5932	0.7494	1.2223	0.8182		0.3770	0.2874
	2	0.3649	1.1286	1.1861	0.6153	1.6253		1.0680	4.6503
	3	0.1522	1.4938	1.5015	0.2027	4.9328		1.4860	13.9067
	4	0.4828		0.4828			0.4828		
8	1	0.4222	0.2640	0.4979	1.6958	0.5897	0.4026		
	2	0.3833	0.7716	0.8616	0.8898	1.1239		0.6697	1.9722
	3	0.2678	1.2066	1.2360	0.4333	2.3076		1.1765	7.4721
	4	0.1122	1.4798	1.4840	0.1512	6.6134		1.4755	16.4334
9	1	0.3700	0.4704	0.5985	1.2365	0.8088		0.2905	0.2480
	2	0.3230	0.9068	0.9626	0.6711	1.4901		0.8473	3.9831
	3	0.2309	1.2634	1.2843	0.3596	2.7811		1.2421	9.0271
	4	0.0860	1.4740	1.4765	0.1165	8.5804		1.4715	18.6849
	5	0.3842		0.3842			0.3842		
10	1	0.3384	0.2101	0.3983	1.6991	0.5885	0.3212		
	2	0.3164	0.6180	0.6943	0.9114	1.0972		0.5309	1.8164
	3	0.2677	0.9852	1.0209	0.5244	1.9068		0.9481	5.9157
	4	0.1849	1.2745	1.2878	0.2871	3.4825		1.2610	10.9284
	5	0.0671	1.4389	1.4405	0.0931	10.7401		1.4373	20.6296

Table XII. Gaussian-to-6 dB Design

ORDER	SECTION	REAL PART	IMAGINARY PART	F_0	α	Q	−3 dB FREQUENCY	PEAKING FREQUENCY	PEAKING LEVEL
3	1	0.9360	1.2168	1.5352	1.2194	0.8201		0.7775	0.2956
	2	0.9360		0.9360			0.9360		
4	1	0.9278	1.6995	1.9363	0.9583	1.0435		1.4239	1.5025
	2	0.9192	0.5560	1.0743	1.7113	0.5844	0.8582		
5	1	0.8075	0.9973	1.2832	1.2585	0.7946		0.5853	0.1921
	2	0.7153	0.2053	0.7442	1.9224	0.5202	0.5065		
	3	0.8131		0.8131			0.8131		
6	1	0.7019	0.4322	0.8243	1.7030	0.5872	0.6627		
	2	0.6667	1.2931	1.4549	0.9165	1.0911		1.1080	1.7809
	3	0.4479	2.1363	2.1827	0.4104	2.4366		2.0888	7.9227
7	1	0.6155	0.7703	0.9860	1.2485	0.8010		0.4632	0.2168
	2	0.5486	1.5154	1.6116	0.6808	1.4689		1.4126	3.8745
	3	0.2905	2.1486	2.1681	0.2680	3.7318		2.1289	11.5169
	4	0.6291		0.6291			0.6291		
8	1	0.5441	0.3358	0.6394	1.7020	0.5876	0.5145		
	2	0.5175	0.9962	1.1226	0.9220	1.0846		0.8512	1.7432
	3	0.4328	1.6100	1.6672	0.5192	1.9260		1.5507	5.9962
	4	0.1978	2.0703	2.0797	0.1902	5.2571		2.0608	14.4545
9	1	0.4961	0.6192	0.7934	1.2505	0.7997		0.3705	0.2116
	2	0.4568	1.2145	1.2976	0.7041	1.4203		1.1253	3.6221
	3	0.3592	1.7429	1.7795	0.4037	2.4771		1.7055	8.0594
	4	0.1489	2.1003	2.1056	0.1414	7.0704		2.0950	17.0107
	5	0.5065		0.5065			0.5065		
10	1	0.4535	0.2794	0.5327	1.7028	0.5873	0.4283		
	2	0.4352	0.8289	0.9362	0.9297	1.0756		0.7055	1.6904
	3	0.3886	1.3448	1.3998	0.5552	1.8011		1.2874	5.4591
	4	0.2908	1.7837	1.8072	0.3218	3.1074		1.7598	9.9618
	5	0.1136	2.0599	2.0630	0.1101	9.0802		2.0568	19.1751

COMPARING THE RESPONSES

The responses of several all-pole filters, namely the Bessel, Butterworth, and Chebyshev (in this case of 0.5 dB ripple), will now be compared. An 8-pole filter is used as the basis for the comparison. The responses have been normalized for a cutoff of 1 Hz. Comparing Figures 12 and 13, it is easy to see the tradeoffs in the various responses. Moving from Bessel through Butterworth to Chebyshev, one can see that the amplitude discrimination improves as the transient behavior gets progressively poorer.

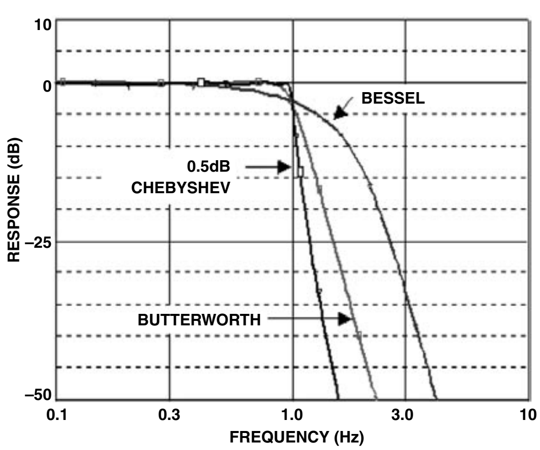


Figure 12. Amplitude Response Comparison

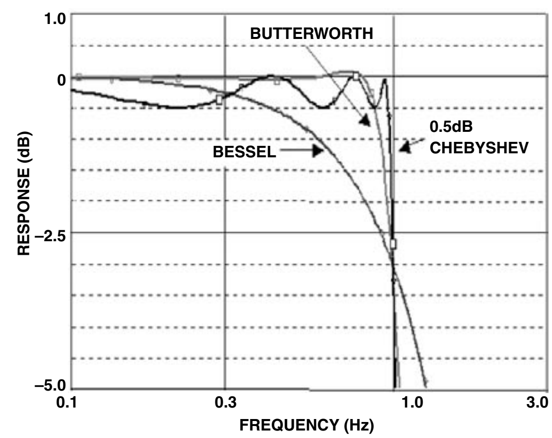


Figure 13. Amplitude Response Comparison (Detail)

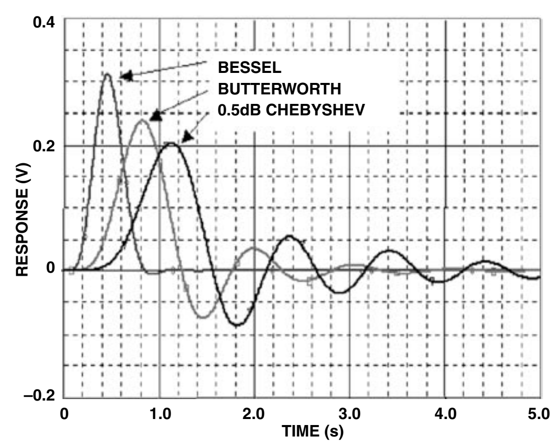


Figure 14. Impulse Response Comparison

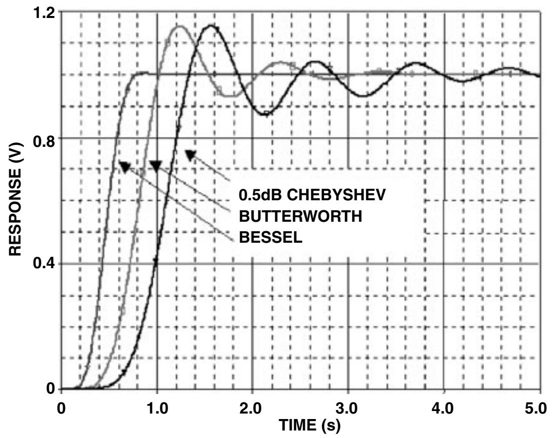


Figure 15. Step Response Comparison

FILTER TOPOLOGIES

Now that it has been decided what to build, it now must be decided how to build it. This means that it is necessary to decide which of the filter topologies to use. Filter design is a two-step process where it is determined what is to be built (the filter transfer function) and then how to build it (the topology used for the circuit). In general, filters are built out of 1-pole sections for real poles, and two pole sections for pole pairs. While one can build a filter out of 3-pole or higher order sections, the interaction between the components in the sections increases and, therefore, so do component sensitivities. It is better to use buffers to isolate the various sections. Additionally, it is assumed that all filter sections are driven from a low impedance source. Any source impedance can be modeled as being in series with the filter input.

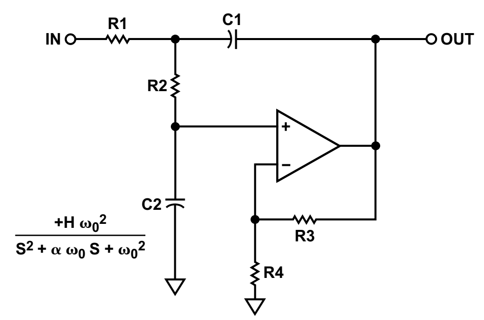
In all of the design equation figures, the following convention will be used:

- H = circuit gain in the pass band or at resonance
- F_C = cutoff or resonant frequency in Hz
- ω_0 = cutoff or resonant frequency in radians/sec
- Q = circuit "quality factor"; indicates circuit peaking
- α = 1/Q = damping ratio

Unfortunately, the symbol α is used for damping ratio. It is not the same as the α used to denote pole locations ($\alpha \pm j\beta$). The same issue occurs for Q. It is used for the circuit quality factor as well as the component quality factor, which are not the same thing. The circuit Q is the amount of peaking in the circuit. This is a function of the angle of the pole to the origin in the s plane. The component Q is the number of losses in what should be lossless reactance. These losses are the parasitics of the components-dissipation factor, leakage resistance, ESR (equivalent series resistance), etc. in capacitors and series resistance and parasitic capacitances in inductors.

SALLEN-KEY FILTER

The Sallen-Key configuration, also known as a voltage control voltage source (VCVS), was first introduced in 1955 by R.P. Sallen and E.L. Key of MIT's Lincoln Labs. It is one of the most widely used filter topologies. One reason for this popularity is that this configuration shows the least dependence of filter performance on the performance of the op amp. This is due to the fact that instead of being configured as an integrator, the op amp is configured as an amplifier, which minimizes its gain-bandwidth requirements. This infers that since the op amp gain-bandwidth product will not limit the performance of the filter as it would if it were configured as an integrator, one can design a higher frequency filter than one can with other topologies. The signal phase through the filter is maintained (noninverting configuration). Another advantage of this configuration is that the ratio of the largest resistor value to the smallest resistor value and the ratio of the largest capacitor value to the smallest capacitor value (component spread) are low, which is good for manufacturability. The frequency and Q terms are somewhat independent, but they are very sensitive to the gain parameter. The Sallen-Key is very Q sensitive to element values, especially for high Q sections. The design equations for the Sallen-Key low-pass filter are shown in Figure 16.



$$\frac{V_O}{V_{IN}} = \frac{H\,\dfrac{1}{R1\,R2\,C1\,C2}}{S^2 + S\left[\left(\dfrac{1}{R1}+\dfrac{1}{R2}\right)\dfrac{1}{C2} + \dfrac{(1-H)}{R2C2}\right] + \dfrac{1}{R1\,R2\,C1\,C2}}$$

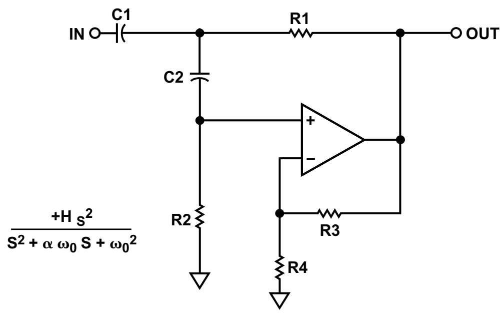
CHOOSE:	C1	R3
THEN:	$k = 2\pi F_0 C1$	$R4 = \dfrac{R3}{(H-1)}$
	$m = \dfrac{\alpha^2}{4} + (H-1)$	
	$C2 = m\,C1$	
	$R1 = \dfrac{2}{\alpha k}$	
	$R2 = \dfrac{\alpha}{2mk}$	

Figure 16. Sallen-Key Low-Pass Design Equations

There is a special case of the second-order Sallen-Key low-pass filter. If the gain is set to 2, the capacitor values, as well as the resistor values, will be the same.

While the Sallen-Key filter is widely used, a serious drawback is that the filter is not easily tuned, due to interaction of the component values on F_0 and Q.

The design equations for the Sallen-Key high-pass filter are shown in Figure 17.



$$\frac{V_O}{V_{IN}} = \frac{H\,s^2}{S^2 + S\left[\dfrac{\dfrac{C2}{R2} + \dfrac{C1}{R2} + (1-H)\dfrac{C2}{R1}}{C1\,C2}\right] + \dfrac{1}{R1\,R2\,C1\,C2}}$$

CHOOSE: C1 R3

THEN: $k = 2\pi F_0 C1$

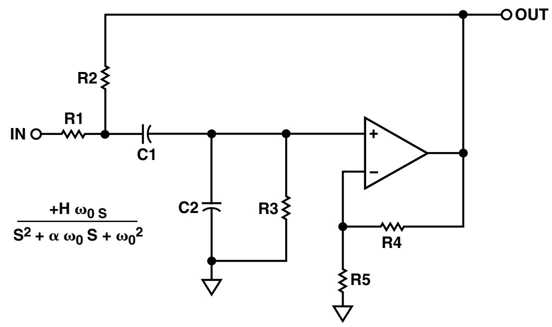
$$R4 = \frac{R3}{(H-1)}$$

$$C2 = C1$$

$$R1 = \frac{\alpha + \sqrt{\alpha^2 + (H-1)}}{4\,k}$$

$$R2 = \frac{4}{\alpha + \sqrt{\alpha^2 + (H-1)}} \times \frac{1}{k}$$

Figure 17. Sallen-Key High-Pass Design Equations



$$\frac{V_O}{V_{IN}} = \frac{H\,s\,\dfrac{1}{R1C2}}{S^2 + S\left[\dfrac{\dfrac{C1}{R3} + \dfrac{(C1+C2)}{R1} + \dfrac{C2}{R2} + \dfrac{C1}{R2}(1-H)}{C1\,C2}\right] + \dfrac{1}{R3\,C1\,C2}\left(\dfrac{R1+R2}{R1\,R2}\right)}$$

CHOOSE: C1 R4

THEN: $k = 2\pi F_0 C1$

$$R5 = \frac{R4}{(H-1)}$$

$$C2 = \frac{1}{2}\,C1$$

$$R1 = \frac{2}{k}$$

$$R2 = \frac{2}{3\,k}$$

$$R3 = \frac{4}{k}$$

$$H = \frac{1}{3}\left(6.5 - \frac{1}{Q}\right)$$

Figure 18. Sallen-Key Band-Pass Design Equations

The band-pass case of the Sallen-Key filter has a limitation. The value of Q will determine the gain of the filter, i.e., it cannot be set independent, as in the low-pass or high-pass cases. The design equations for the Sallen-Key band-pass are shown in Figure 18.

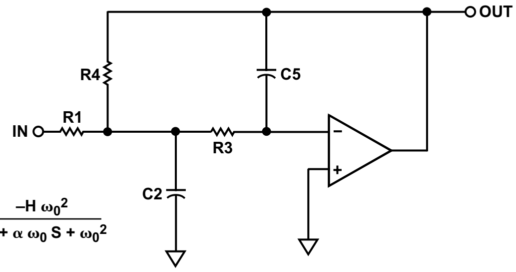
MULTIPLE FEEDBACK FILTER

The multiple feedback filter uses an op amp as an integrator. Therefore, the dependence of the transfer function on the op amp parameters is greater than in the Sallen-Key realization. It is hard to generate high Q, high frequency sections due to the limitations of the open-loop gain of the op amp. The open-loop gain of the op amp should be at least 20 dB (×10) above the amplitude response at the resonant (or cutoff) frequency, including the peaking caused by the Q of the filter. The peaking due to Q will cause an amplitude, A_0:

$$A_0 = H\,Q \qquad (2)$$

where H is the gain of the circuit. The multiple feedback filter will invert the phase of the signal. This is equivalent to adding the resulting 180° phase shift to the phase shift of the filter itself.

The maximum to minimum component value ratios are higher in the multiple feedback case than in the Sallen-Key realization. The design equations for the multiple feedback low-pass filter are given in Figure 19.



$$\frac{-H\,\omega_0^2}{S^2 + \alpha\,\omega_0\,S + \omega_0^2}$$

$$\frac{V_O}{V_{IN}} = \frac{-H\,\frac{1}{R1\,R3\,C2\,C5}}{S^2 + S\,\frac{1}{C2}\left(\frac{1}{R1} + \frac{1}{R3} + \frac{1}{R4}\right) + \frac{1}{R3\,R4\,C2\,C5}}$$

CHOOSE: C5

THEN: $k = 2\,\pi\,F_0 C5$

$$C2 = \frac{4}{\alpha^2}\,(H + 1)\,C5$$

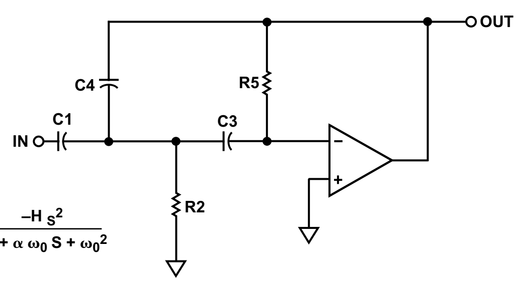
$$R1 = \frac{\alpha}{2\,H\,k}$$

$$R3 = \frac{\alpha}{2\,(H + 1)k}$$

$$R4 = \frac{\alpha}{2\,k}$$

Figure 19. Multiple Feedback Low-Pass Design Equations

The design equations for the multiple feedback high-pass filter are given in Figure 20.



$$\frac{-H\,s^2}{S^2 + \alpha\,\omega_0\,S + \omega_0^2}$$

$$\frac{V_O}{V_{IN}} = \frac{-S^2\,\frac{C1}{C4}}{S^2 + S\,\frac{(C1 + C3 + C4)}{C3\,C4\,R5} + \frac{1}{R2\,R5\,C3\,C4}}$$

CHOOSE: C1

THEN: $k = 2\,\pi\,F_0 C1$

$C3 = C1$

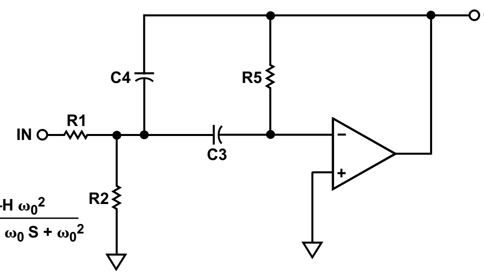
$$C4 = \frac{C1}{H}$$

$$R2 = \frac{\alpha}{k\left(2 + \frac{1}{H}\right)}$$

$$R5 = \frac{H\left(2 + \frac{1}{H}\right)}{\alpha k}$$

Figure 20. Multiple Feedback High-Pass Design Equations

The design equations for the multiple feedback band-pass case are given in Figure 21. This circuit is widely used in low Q (<20) applications. It allows some tuning of the resonant frequency, F_0, by making R2 variable. Q can be adjusted (with R5) as well, but this will also change F_0. Tuning of F_0 can be accomplished by monitoring the output of the filter with the horizontal channel of an oscilloscope, with the input to the filter connected to the vertical channel. The display will be a Lissajous pattern consisting of an ellipse that will collapse into a straight line at resonance, since the phase shift will be 180°. One could also adjust the output for maximum output, which will also occur at resonance; this is usually not as precise, especially at lower values of Q, where there is a less pronounced peak.



$$\frac{-H\,\omega_0^2}{S^2 + \alpha\,\omega_0\,S + \omega_0^2}$$

$$\frac{V_O}{V_{IN}} = \frac{-S\,\frac{1}{R1\,C4}}{S^2 + S\,\frac{(C3 + C4)}{C3\,C4\,R5} + \frac{1}{R5\,C3\,C4}\left(\frac{1}{R1} + \frac{1}{R2}\right)}$$

CHOOSE: C3

THEN: $k = 2\,\pi\,F_0 C3$

$C4 = C3$

$$R1 = \frac{1}{Hk}$$

$$R2 = \frac{1}{(2Q - H)k}$$

$$R5 = \frac{2Q}{k}$$

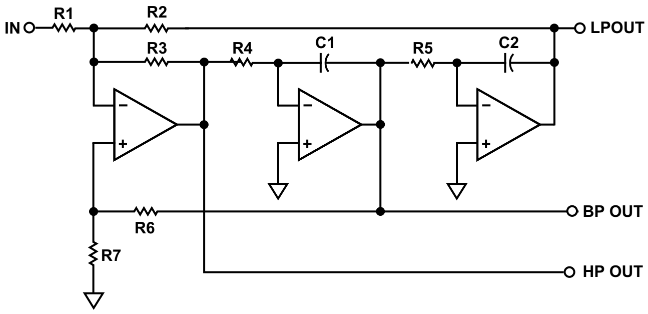
Figure 21. Multiple Feedback Band-Pass Design Equations

STATE VARIABLE FILTER

The state-variable realization offers the most precise implementation at the expense of many more circuit elements. All three major parameters (gain, Q, and ω_0) may be adjusted independently, and low-pass, high-pass, and band-pass outputs are available simultaneously. Note that the low-pass and high-pass outputs are inverted in phase while the band-pass output maintains the phase. The gain of each of the outputs of the filter is also independently variable.

Since all parameters of the state variable filter can be adjusted independently, component spread can be minimized. Also, variations due to temperature and component tolerances are minimized. The op amps used in the integrator sections will have the same limitations on op amp gain-bandwidth as described in the multiple feedback section.

The design equations for the state variable filter are shown in Figure 22.



$$A_{LP(s = 0)} = -\frac{R2}{R1}$$

$$A_{HP(s = \infty)} = -\frac{R3}{R1}$$

$$\omega_0 = \sqrt{\frac{R3}{R2\ R4\ R5\ C1\ C2}}$$

LET R4 = R5 = R, C1 = C2 = C

$$A_{BP(s = \omega_0)} = \frac{\frac{R6 + R7}{R7}}{R1\left(\frac{1}{R1} + \frac{1}{R2} + \frac{1}{R3}\right)}$$

CHOOSE R1:

$$R2 = A_{LP}\ R1$$

$$R3 = A_{HP}\ R1$$

CHOOSE C:

$$R = \frac{2\ \pi\ F_0}{C}\sqrt{\frac{A_{HP}}{A_{LP}}}$$

CHOOSE R7:

$$R6 = R7\sqrt{R2\ R3}\ Q\left(\frac{1}{\frac{1}{R1} + \frac{1}{R2} + \frac{1}{R3}}\right)$$

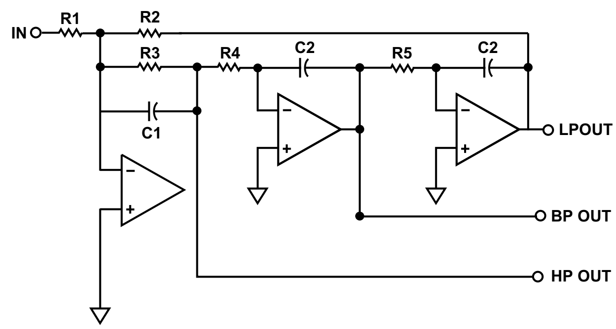
Figure 22. State Variable Design Equations

Tuning the resonant frequency of a state variable filter is accomplished by varying R4 and R5. While both do not have to be tuned, it is generally preferable if varying over a wide range. Holding R1 constant, tuning R2 sets the low-pass gain and tuning R3 sets the high-pass gain. Bandpass gain and Q are set by the ratio of R6 and R7.

Since the parameters of a state variable filter are independent and tunable, it is easy to add electronic control of frequency, Q and ω_0. This adjustment is accomplished by using multiplying DACs (MDACs) or digital potentiometers. For the integrator sections, adding the MDAC effectively increases the time constant by dividing the voltage driving the resistor, which, in turn, provides the charging current for the integrator capacitor. In effect, this raises the resistance and, in turn, the time constant. The Q and gain can be varied by changing the ratio of the various feedback paths. A digital potentiometer will accomplish the same feat in a more direct manner, by directly changing the resistance value. The resulting tunable filter offers a great deal of utility in measurement and control circuitry.

BIQUADRATIC (BIQUAD) FILTER

A close cousin of the state variable filter is the biquad. The name of this circuit, first used by J. Tow in 1968 and later by L.C. Thomas in 1971, is derived from the fact that the transfer function is quadratic in both the numerator and the denominator. Therefore, the transfer function is a biquadratic function. This circuit is a slight rearrangement of the state variable circuit. One significant difference is that there is not a separate high-pass output. The band-pass output inverts the phase. There are two low-pass outputs, one in phase and one out of phase. With the addition of a fourth amplifier section, a high-pass filter may be realized. The design equations for the biquad are given in Figure 23.



CHOOSE C, R2, R5

$$K = 2\ \pi\ f_0\ C$$

$$C1 = C2 = C$$

$$R1 = \frac{R2}{H}$$

$$R3 = \frac{1}{k\ \alpha}$$

$$R4 = \frac{1}{k^2\ R2}$$

CHOOSE C, R5, R7

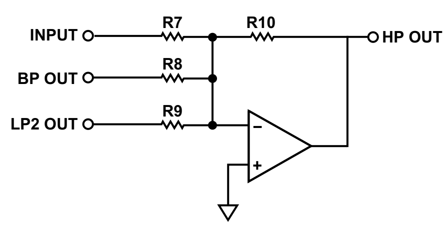
$$K = 2\ \pi\ f_0\ C$$

$$C1 = C2 = C$$

$$R1 = R2 = R3 = \frac{1}{k\ \alpha}$$

$$R4 = \frac{1}{k^2\ R2}$$

$$R6 = R5$$



$$R7 = R8 = R9 = R$$

$$R10 = \frac{R}{H}$$

Figure 23. Biquad Design Equations

Referring to Figure 23, the input, band-pass, and second low-pass outputs are summed for the high-pass output. In this case, the constraints are that R1 = R2 = R3 and R7 = R8 = R9.

Like the state variable, the biquad filter is tunable. Adjusting R3 will adjust the Q. Adjusting R4 will set the resonant frequency. Adjusting R1 will set the gain. Frequency would generally be adjusted first, followed by Q and then gain. Setting the parameters in this manner minimizes the effects of component value interaction.

OP AMP REQUIREMENTS

The curves that were generated for the prototype responses were done using an ideal op amp. In reality, the op amp is a single-pole low-pass filter. The amplifier's dominant pole is the corner frequency of the filter. The op amp's transfer function would be added to the filter response.

In practice, this means that if the gain-bandwidth product of the op amp is not at least an order of magnitude greater than the cutoff frequency of the filter, there will be some interaction. If the gain-bandwidth product of the amplifier is more than an order of magnitude higher than the filter, the response of the op amp can generally be ignored. In any case, if there is concern, the filter SPICE deck can be downloaded and the SPICE model of the specific op amp that will be used can be simulated with SPICE.

A current feedback amplifier can only be used with Sallen-Key topology since this is the only topology in which the op amp is configured as an amplifier. In the other topologies, the op amp is used with capacitors in the feedback network, which is inappropriate for the current feedback amplifier.

Another choice is that of bipolar or FET input devices. In general, if the impedance level of the filter is less than 1 kΩ, a bipolar op amp is the appropriate choice. If the impedance is greater than 10 kΩ, a FET input op amp is a better choice. This is entirely due to the FET amps having higher input impedance, which will be less of a load to the network.

A final word of caution: filters with high Q sections can cause the dynamic range of the op amp to be exceeded. This is due to peaking of the section. The peaking due to Q will cause an amplitude, A_0:

$$A_0 = H\,Q$$

where H is the gain of the circuit.

Also remember:

$$\alpha = \frac{1}{Q}$$

AN EXAMPLE

As an example, an antialiasing filter will now be designed.

The specifications for the filter are as follows:

1. The cutoff frequency is 8 kHz.
2. The stop-band attenuation is 72 dB. This corresponds to a 12-bit system.
3. The stop-band frequency is 42 kSPS. This assumes a 100 kSPS A/D converter. The Nyquist frequency is 50 kHz. Subtracting 8 kHz, for the image of the pass band around the sample rate gives us 42 kHz.
4. The Butterworth filter response is chosen in order to give the best compromise between attenuation and phase response.

Taking the Butterworth curves (Figure 1), a horizontal line is drawn at 72 dB. A vertical line is drawn at 5.25 Hz. This is the ratio of F_S/F_0 (see Figure 24). This shows that a filter order of 5 is required. This information is then used for input to the filter tool.

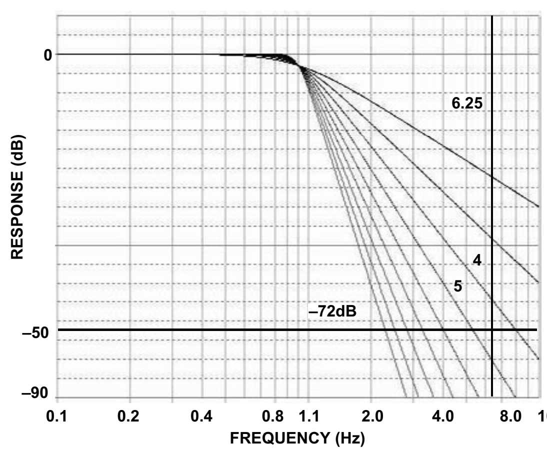


Figure 24. Determining Filter Order

USING THE TOOL

First, the filter order must be determined. To do this, use the filter response curves as described in Figure 24.

This information is then used in the design tool. First, the filter response is entered. In this example, Lowpass is selected. The other options are shown.

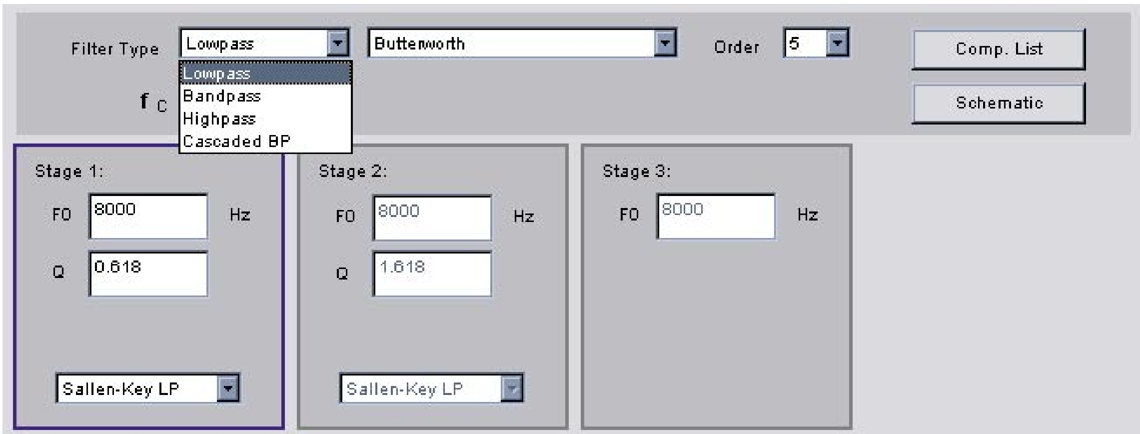


Figure 25. Entering the Response Type

Next, the response shape is entered. Butterworth was chosen.

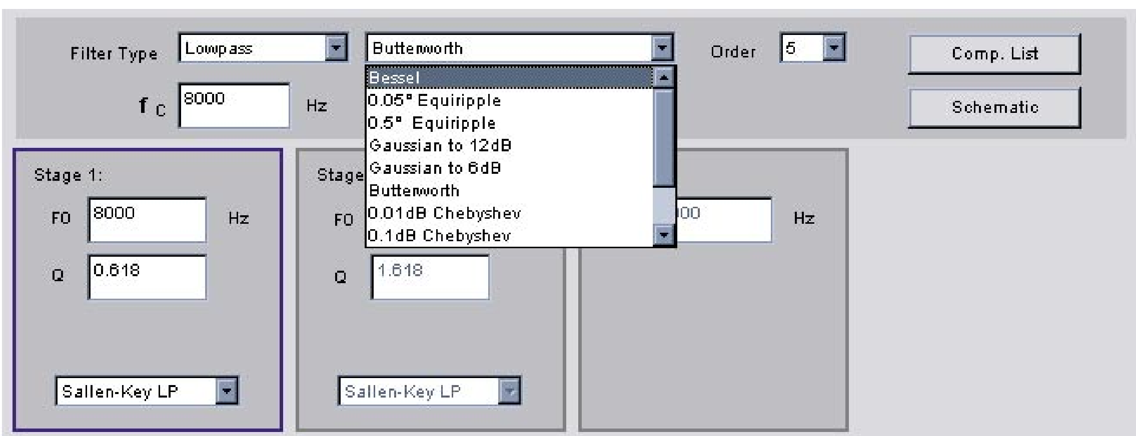


Figure 26. Filter Response

The filter order is then entered. The choices are 1 to 8. In this example, fifth-order is chosen. The cutoff (or center) frequency is also entered.

A fifth-order filter is made up of two second-order sections and a single-pole section. This is shown in the next row of boxes in the tool. For each of the second-order sections, the topology of the filter is entered. The active box is outlined.

The sections can be configured in any order. Typically, the low Q sections are put first. This may help with the problem of exceeding the dynamic range of the filter by providing some attenuation before the peaking of the higher Q section.

The filter tool has entered the appropriate center frequency and Q for each second-order section, and the center frequency for the single pole. Note that for custom filters, these values can be entered manually. The filter topology is then entered from the choices given. Each section is separate (Figure 27).

The details of the individual section are then entered. Since the actual component values are ratiometric, one value must be specified, and the rest will then be determined. Setting the capacitor was chosen since there is typically less freedom in selecting a value than for a resistor (Figure 28).

The gain is entered next. This typically requires the user to enter another component value. For the specific example of a unity gain Sallen-Key section, the value of feedback resistor can be specified as 0 Ω. There is some error detection on the component entry. For the state variable and biquad section, other resistor values are entered.

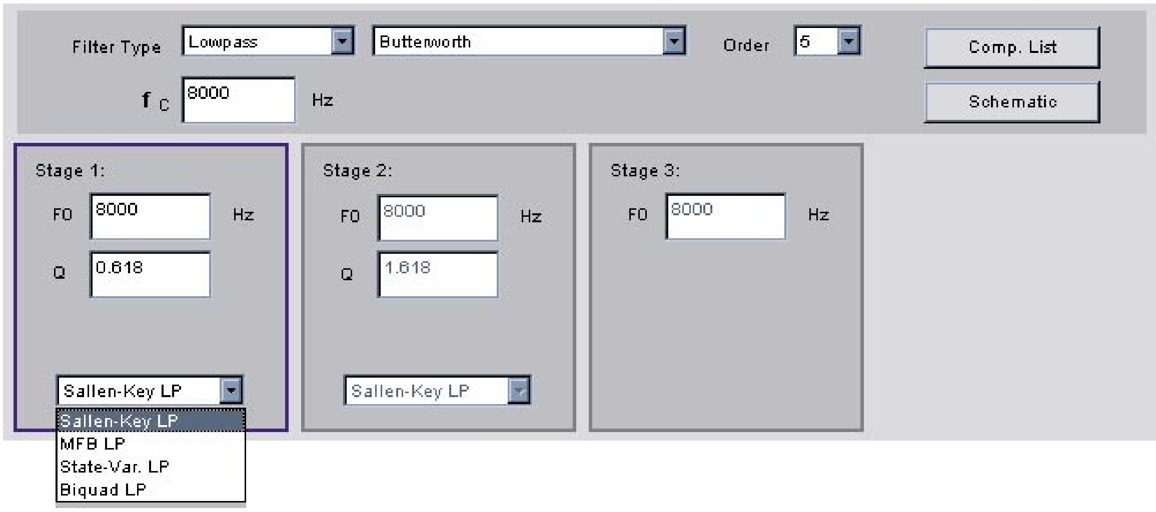


Figure 27. Filter Topology

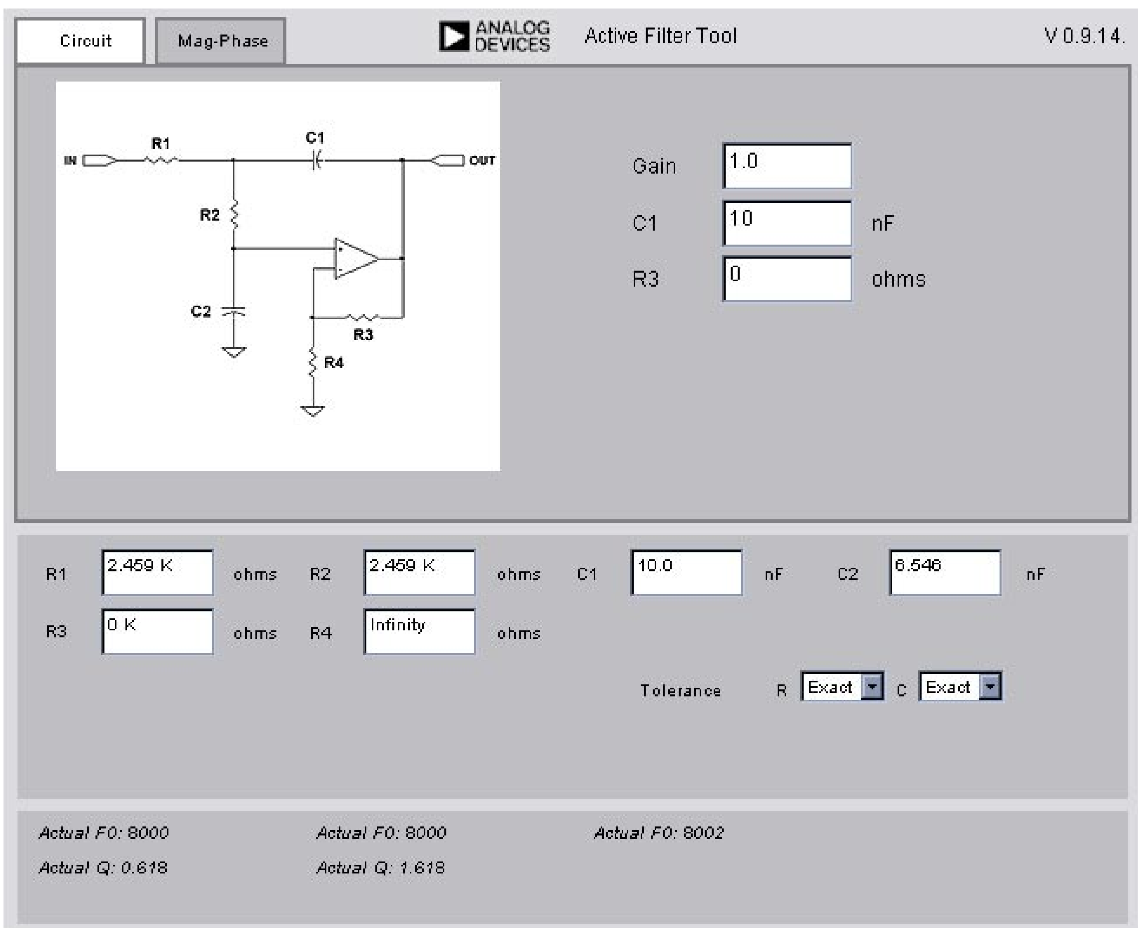


Figure 28. Component Values (Exact)

These values are the exact calculated values. The filter will typically be built with standard value components. The tool allows the user to specify the component tolerances. When this is done, the standard values are substituted in the boxes. Additionally, the percent error in F_0 and Q caused by the change in value is calculated.

For custom filters, the values can be entered manually.

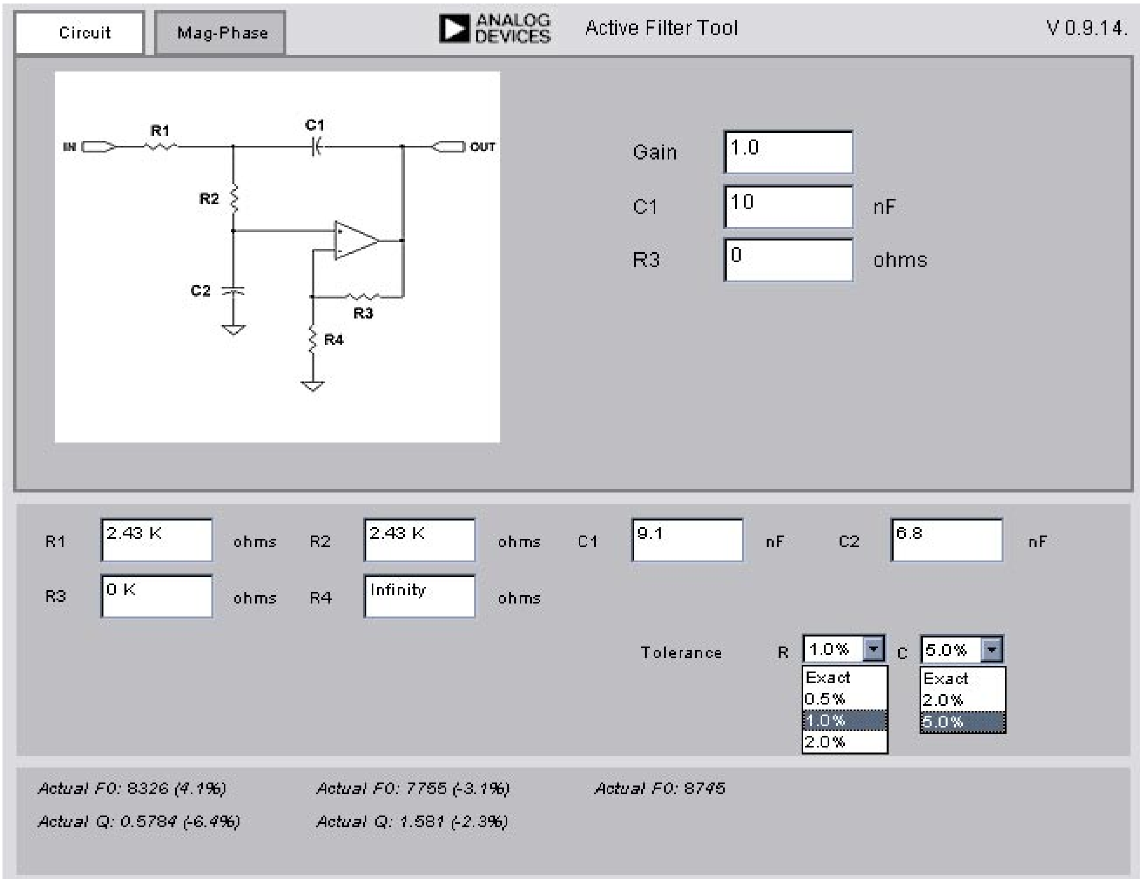


Figure 29. Component Values (Standard)

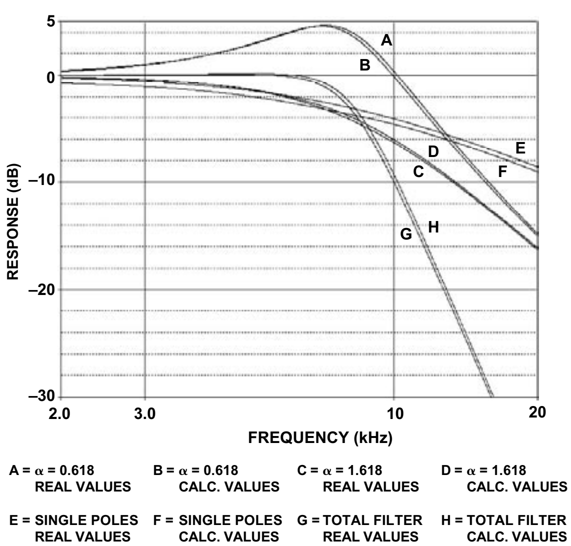


Figure 30. Filter Response Change Due to Standard Values

Figure 30 shows the change in the filter response due to using standard values versus exact values. Whether this is acceptable is a decision that the designer must make.

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